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At Right Angles

A RESOURCE FOR SCHOOL MATHEMATICS

ISSN 2582-1873

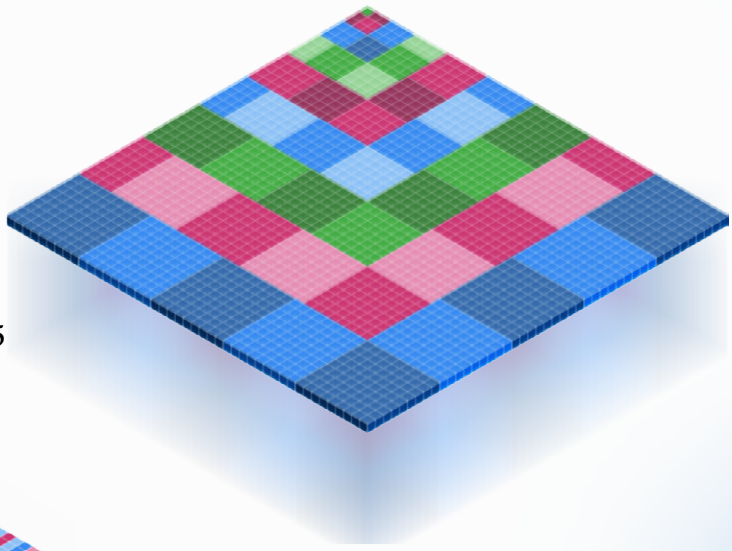
SEEING 2025 THROUGH ART AND MATHEMATICS

PULLOUT
PATTERNS & PRE-ALGEBRA

Seeing 2025 through Art and Mathematics

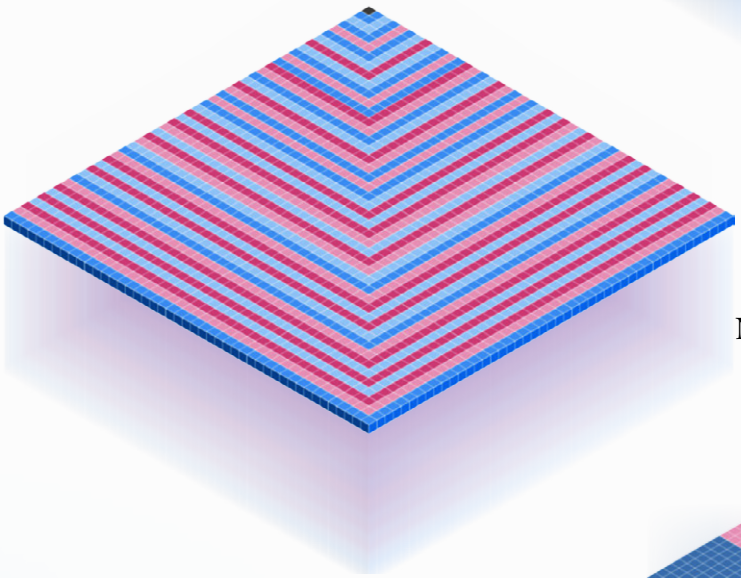
$(1+2+3+4+5+6+7+8+9)^2 = 2025$

The square of the sum of the natural numbers from **1 to 9**



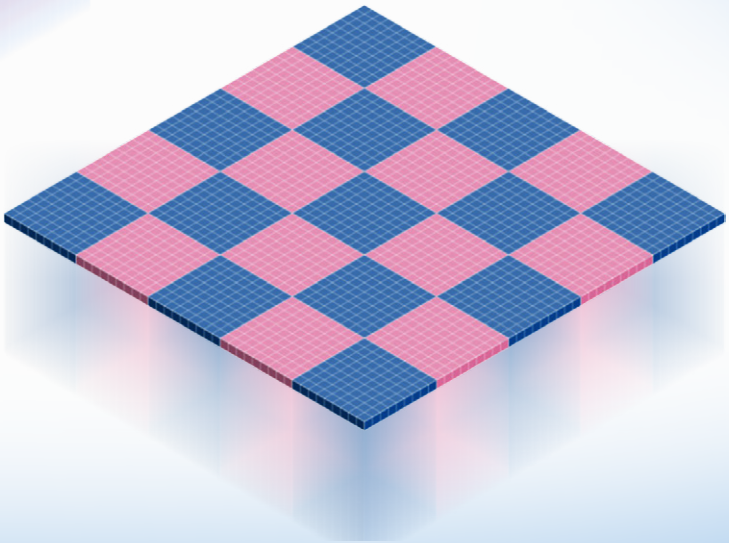
$(1 + 3 + 5 + \ldots\ldots+89) = 2025$

The sum of the odd numbers from **1 to 89 = 2025**



$9^2 \times 5^2 = 2025$

Nine squares repeated twenty five times is **2025**



2025 began with a spate of WhatsApp forwards on how many ways 2025 could be represented with mathematical equations. And editor Swati Sircar visualized these for our readers! We hope that your students are inspired to make more such colourful visuals and we encourage you to find patterns in the colouring too!

From the Editors Desk . . .

Dear Readers,

The first issue of the year 2025! Not so new, now that we are three months, a quarter of the year and one winter away from the old! In the break between the November 2024 and March 2025 issue, Padmapriya Shirali and I spent a few days in some of the schools in Chamarajanagar district (Karnataka) and Bhopal and Damoh districts (Madhya Pradesh) that the Azim Premji Foundation interacts with. Our return to classrooms and our interactions with teachers and students were delightful and thought provoking and we came back buzzing with ideas and determined to build on the energy and enthusiasm of the children and their teachers. Editors Rudresh and Sandeep Diwakar coordinated our visits and it was heartening to see how teachers and students welcomed the Foundation's presence in the classroom and how deeply our Resource Persons were connected with the Field. We plan to do this more often – you will be seeing a lot of responses to questions, ideas for articles and suggestions to address misconceptions in this and upcoming issues of At Right Angles.

In the March 2024 issue, we had covered the features of the new class 1 and class 2 mathematics textbooks. We open this issue with a look at how these have been received and used in classrooms across India. How has this influenced the pedagogy? How easy has the material been to transact? What are the welcome changes? What is missing? With detailed input from teacher Sonia Kundu, Kshama Chakravarthy bases this article on several conversations with teachers, reports on classroom transactions and feedback from a painstaking survey conducted across states from Karnataka to Uttarakhand.

In the Classroom section, we bring you several vignettes from classrooms – planning of lessons and facilitated discussions by Rahul Singh Rathore and Jagrati Mehra that led to deeper conceptual understanding of Circles and Patterns. Shekh Mohammed Zahid shares insights on crafting questions that are deeper, assessing both a student's understanding as well as misconceptions – several examples are given along with some guidelines on how to do this in other topics. Kshama completes her series on Montessori materials – their features, the learning intentions – you are also provided with links to Math Space which teach us how to develop low cost versions of these.

Number Wheels and a modified Uno Game! What more do you need to explore the Joy of Mathematics! Have fun reading and experimenting and if you develop versions that lead to more learning through play, observation, discussion and documentation, do send in your pictures and ideas to us.

Children's fiction with a mathematical angle – Manisha Goyal describes a series by a Japanese writer (Anno) that can be plumbed for mathematical content and she describes in detail how she did just that in her classrooms. There are many more ideas in the Review, and I would encourage you to read the story online. His visuals are so rich, each of them can help your students understand counting, place value, mathematical operations, budgeting, planning and many more useful life skills!

We also do a comparative review this time – materials that are used to teach numeracy, how and when to use their features, these are systematically listed by Math Space.

The Pullout was inspired by Resource Person Sowmya N from Chamarajanagar who asked Padmapriya for ideas on introducing Algebra. We hope you benefit from her query and we encourage you all to share such requests.

Send your feedback, questions, ideas to AtRightAngles.editor@apu.edu.in We would love to hear from you.

Warm regards,

Sneha Titus
Chief Editor, At Right Angles.

At Right Angles is a publication of Azim Premji University. It aims to reach out to teachers, teacher educators, students & those who are passionate about mathematics. It provides a platform for the expression of varied opinions & perspectives and encourages new and informed positions, thought-provoking points of view and stories of innovation. The approach is a balance between being an 'academic' and 'practitioner' oriented magazine.

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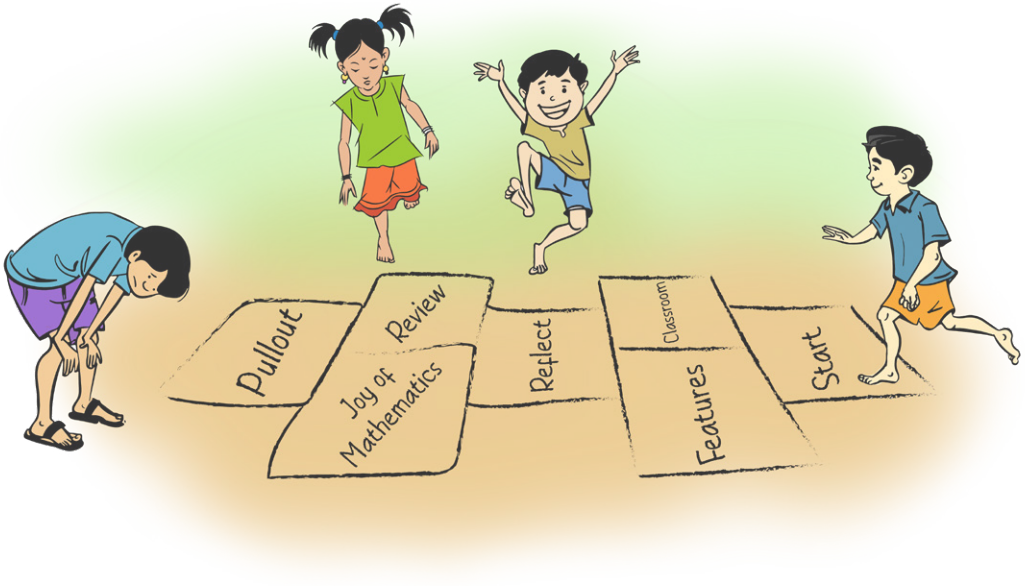
Pullout



A Review of the New NCERT Math Textbooks for Grades 1 and 2

Kshama Chakravarthy & Sonia Kundu

This article examines the new NCERT Math textbooks for grades 1 and 2, comparing them with the vision outlined in the NCF-FS. Some teachers also share their experiences and feedback, followed by a summary of insights from about 90 teachers across Madhya Pradesh, Uttarakhand, Chhattisgarh, Maharashtra, and Karnataka regarding the updated textbooks.

The new NCERT Grade 1 and 2 textbooks are noticeably more vibrant, structured, well-organized, and holistic than their previous versions. The foreword and "About the Book" sections effectively communicate the context and objectives behind the revisions.

> *Based on the principles and objectives enunciated under NEP 2020 — as well as on research from a range of disciplines (including neuroscience and early childhood education), on experiences and accumulated knowledge from the ground, and on the aspirations and goals of our Nation — the National Curriculum Framework for Foundational Stage (NCF-FS) was developed and released on 22 October 2022. Subsequently, textbooks have been developed to bring to life the curricular approach of the NCF-FS. The textbooks attempt to connect to the children's real life by recognising their learning in the classroom and the significant learning resources in the family and the community.*
>
> - Joyful Mathematics, 2023

The textbooks incorporate a wide range of themes, including subtly highlighting the relevance of elders (especially grandparents) and inclusivity of differently abled individuals, sensitivity towards nature, awareness of social issues, and the promotion of values such as sharing, curiosity, and observation skills. Aligned with the NCF-FS's play-based approach to learning, the *Joyful Mathematics* textbooks for grades 1 and 2 feature numerous activities designed to be conducted both inside and outside the classroom, supporting the goal of experiential learning for holistic development.

Mathematical concepts have been introduced through clear, contextual illustrations. These images support comprehension and also enhance children's visual and reading skills. Specific examples, accompanied by images, are provided later in the article. Oral discussions with children have been integrated into the chapters to encourage them to verbalize and express their thought processes. The books have been designed to be text-cum-workbooks, to offer children opportunities to draw, colour, and write. However, some teachers note that the space for solving problems, typically found in a standard workbook, is somewhat limited.

Keywords: NCERT textbooks, review, survey, new version, interview, user experience

"There is no space for completing activities. It should either be provided in the textbook, or if it becomes bulky, a separate workbook with questions."

- Garima Bhatt, Azim Premji School, Uddham Singh Nagar, Uttarakhand

About the amount of practice itself, about 13% of the teachers who participated in the survey (the summary of which is shared in the end) want to reduce the amount of practice, while 48% of them want to increase practice questions. Some teachers mentioned the need for more practice for specific reasons.

"There is a need for highlighting the importance of practising procedural aspects of mathematics at this stage. Textbooks could include first, importance of practice with reasons and also include specific suggestions, resources for teachers to give that practice for their students.

There is also no mention of Fact fluency in the textbooks and creating Fact sheets at the end of the textbook such as addition facts and subtraction facts could help students develop fact fluency which might get missed when conceptual understanding is overemphasised."

- Anagh, Azim Premji School, Bengaluru, Karnataka

"For a diverse class with varied learning levels, it's good to have different levels of questions for practice. I understand that a textbook has its own limitations and cannot fulfill everyone's demands but it can be better if it includes more practice questions of different levels. For each activity/concept, there is a description, pictures and then the 'Let us do' section which includes 3-4 questions based on the activity, which can be increased. Kids usually enjoy solving the textbook more than working in the notebook or worksheets."

- Aakanksha, Azim Premji School, Barmer, Rajasthan

The books have various activities that are intended as suggestions. The idea is to encourage teachers to create their own activities and supplement them with local toys, games, or materials found in the child's immediate environment, to facilitate hands-on learning with concrete objects. Teachers have the flexibility to adapt, modify, and tailor the activities according to their specific context, as long as the focus remains on developing the key competencies for children at this stage. The extent to which a teacher can create a world-class learning experience, aligned with the vision outlined in the NCF-FS document, depends on factors such as teacher capability, intent, availability of resources, and mentorship. According to the survey we conducted, 80% of teachers reported conducting additional activities, such as simulating a shopping experience with tokens and fake money, counting with fingers, stones, counters, and dice, or exploring the concept of heavy and light through hands-on experiences. These efforts are certainly a step in the right direction!

Now, let's take a look at the chapter structure. Each chapter (in both grades) provides regular instructions on what teachers should do (Figure 1), along with talking points and opportunities for discussion on varied topics such as trains, flags, animals, sharing, Ekta Diwas, balanced diets, and the ill effects of overeating.

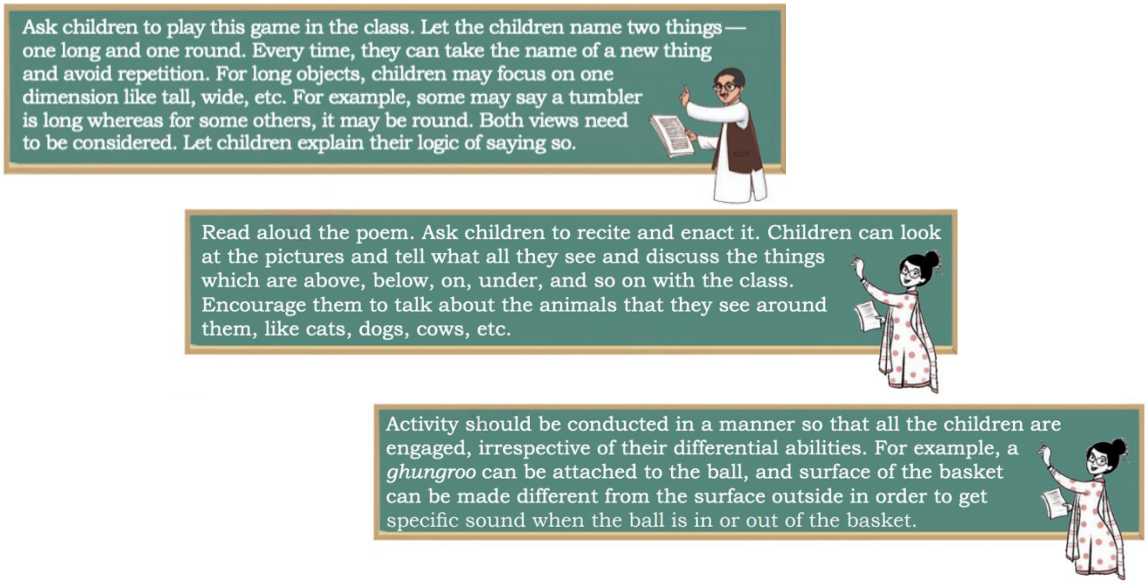


Figure 1: Instructions for teachers (Grade 1, Chapter 2, Page 10; Grade 1, Chapter 1, Page 2 and 4)

There are activities designed to expand vocabulary, promote national integration, and foster patriotism (Figure 2). Sections like "Think and Answer" serve as comprehension checks, while game ideas (such as finding a hidden object through verbal instructions or throwing a ball into a basket) and poems (with follow-up questions for understanding) encourage active participation. In one activity, children line up like a train, reciting a rhyme and identifying relative positions (Figure 3).

Figure 2: Discussion around the national flag (Grade 1, Chapter 1, Page 6)

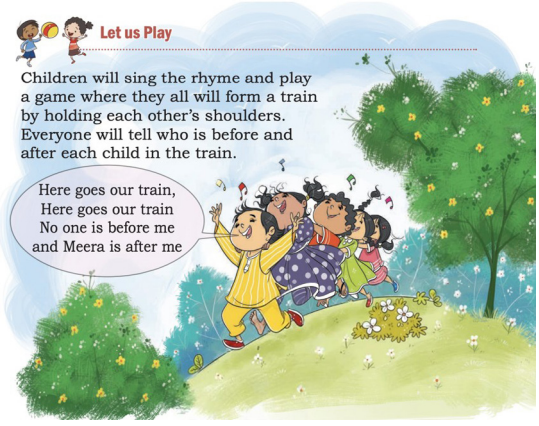


Figure 3: The train game (Grade 1, Chapter 1, Page 8)

Project work is another key element in the new textbooks which encourages students to apply mathematical concepts in creative and practical ways. It promotes experiential learning by connecting math to everyday life, encourages teamwork, develops critical thinking and problem-solving skills.

Chapters conclude with a project or real-world problem-solving activity like, measuring items at home, creating number patterns, or measuring the quantities etc.

- Sonia Kundu, APS, Uttarkashi, Uttarakhand

Along with project work (such as making a list of items to buy and noting down their costs while shopping) (Figure 4a), chapters include fun activities (like showing the number 3 or 4 in different ways with fingers) (Figure 4b), and an introduction to various cultures and places (such as Dal Lake in Kashmir, shadow games from Karnataka, Garba from Gujarat and snake boat race from Kerala) (Figure 4c).

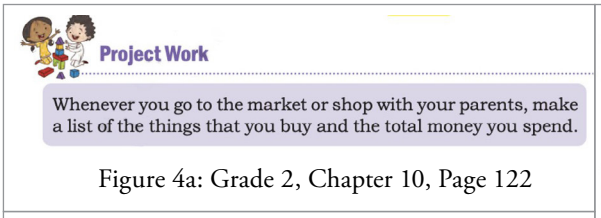


Figure 4a: Grade 2, Chapter 10, Page 122

Figure 4b: Grade 1, Chapter 3, Page 21

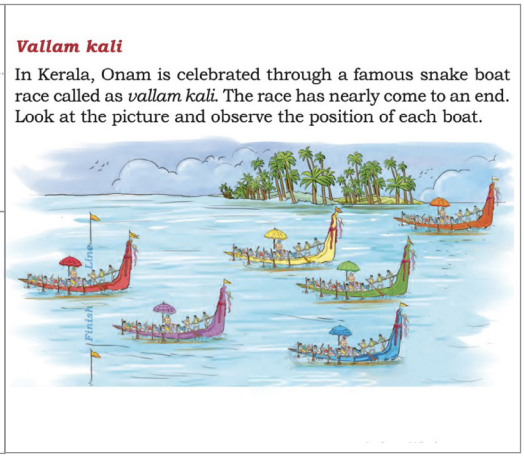


Figure 4c: Grade 2, Chapter 1, Page 14

The start of each chapter has a QR code to scan on the top. It helps the teacher access e-resources such as audios, videos, multimedia, texts etc. related to the themes given in the chapter. For example, the QR code in Chapter 2, Grade 1 (What's Long What's Short) takes one to an audio resource (the entire chapter read out) and a video resource that has the story of "Wise Grandmother" in cartoon form, followed by questions and exercises that are covered in the chapter, in an interactive format.

Figure 5: Grade 1, Chapter 2, Page 10 and 14

60% of the teachers surveyed claim to have made use of this digital content. Teachers have mentioned that the activities and the audio resources are beneficial and make learning easier. The additional resources give them the confidence to explain correctly. However, a couple of teachers mentioned that phones/ internet are not allowed in the school and so they are not able to use this facility while in school.

A few teachers were interviewed to get their perspective on the textbooks, its implementation and their experience with students. We present this to you in the form of a Whatsapp chat, telephonic interview and a documented report of the conversations.

Kshama Chakravarthy's WhatsApp Chat with Garima Bhatt, Teacher at Azim Premji School, Uddham Singh Nagar, Uttarakhand, Teaches Grade 2-5

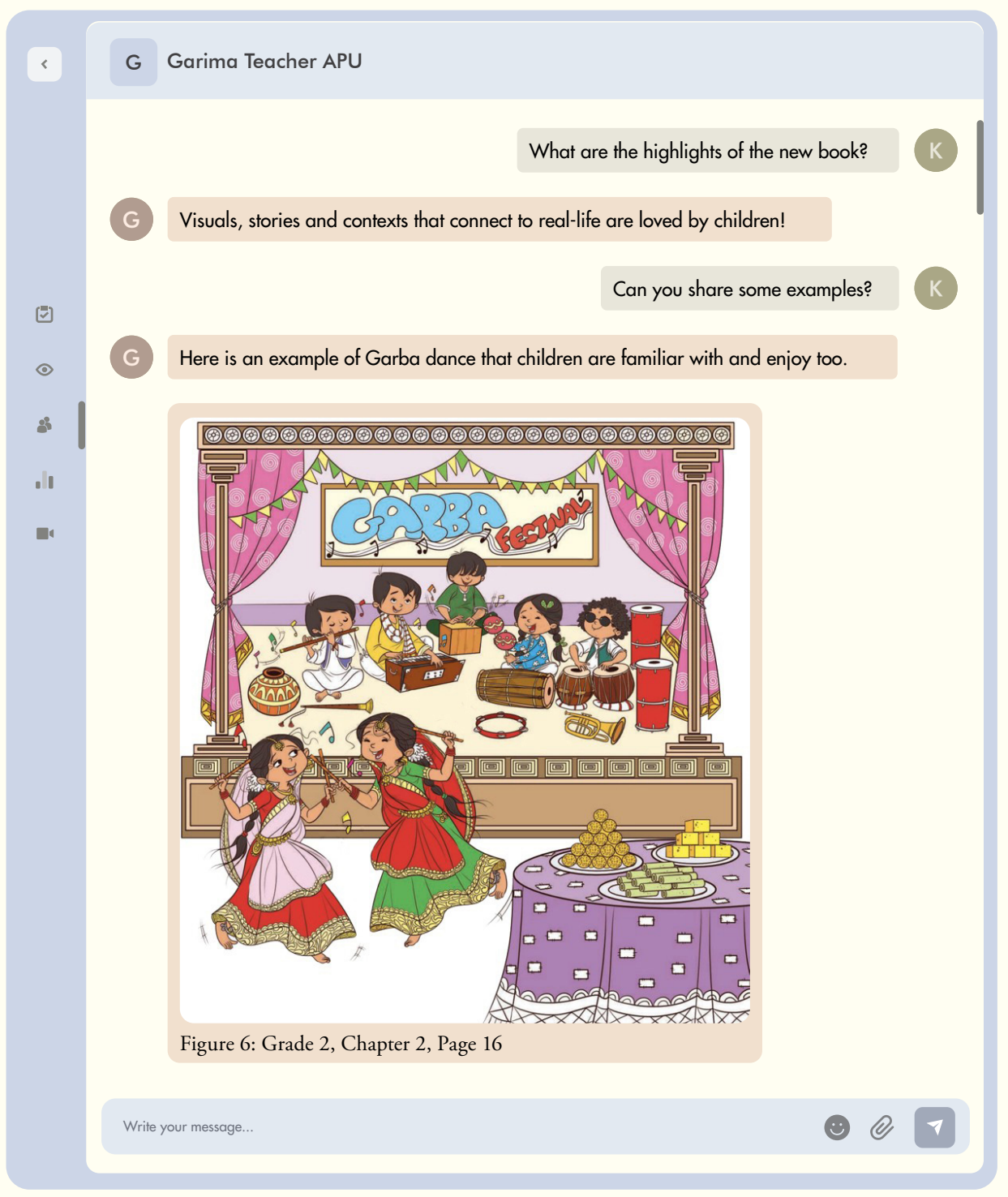


Figure 6: Grade 2, Chapter 2, Page 16

G Garima Teacher APU

G In the first chapter-maala out of seashells is interesting for them. Weighing laddus or other items on the weighing scale is something that kids relate well to as they have seen it in the vegetable market or their family members may even own such scales if they sell these items.

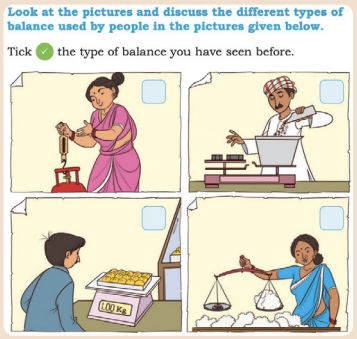


Figure 7: Grade 2, Chapter 7, Page 79

Figure 8: Grade 2, Chapter 9, Page 104. The chapter on Time has a nice picture story.

Thank you! Do you have any suggestions for improvement? K

G Yes. The order of chapters in Grade 2 can be altered. Currently Chapter 1 is on numbers, Chapter 2 on 3D shapes, Chapter 3 on numbers again, Chapter 4 on 2D shapes and then it moves to lines. It may be better to complete numbers first, then 3D shapes and then 2D.

Do you think there's a specific reason for the current flow? K

G The current flow may be done to break the monotony and help some kids who are finding numbers overwhelming, but my personal opinion is that it is better that kids build on what they have learnt, and take to completion the understanding of numbers, before switching to other topics.

All right. Any final words? K

G Overall this is a really nice book and a great effort!

Write your message...

Kshama Chakravarthy's Phone Conversation with Poonam, Teacher at Azim Premji School, Uddham Singh Nagar, Uttarakhand, Teaches Grades 1 and 2

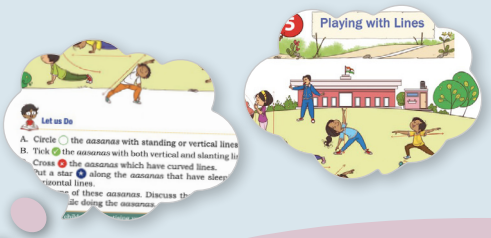
What do you/children like in this new book?

I love the pictures, examples, stories, project work, activities and activity ideas. The puzzles are a big hit among students in both the grades! This new version has a good mix of real-life examples and TLMs.

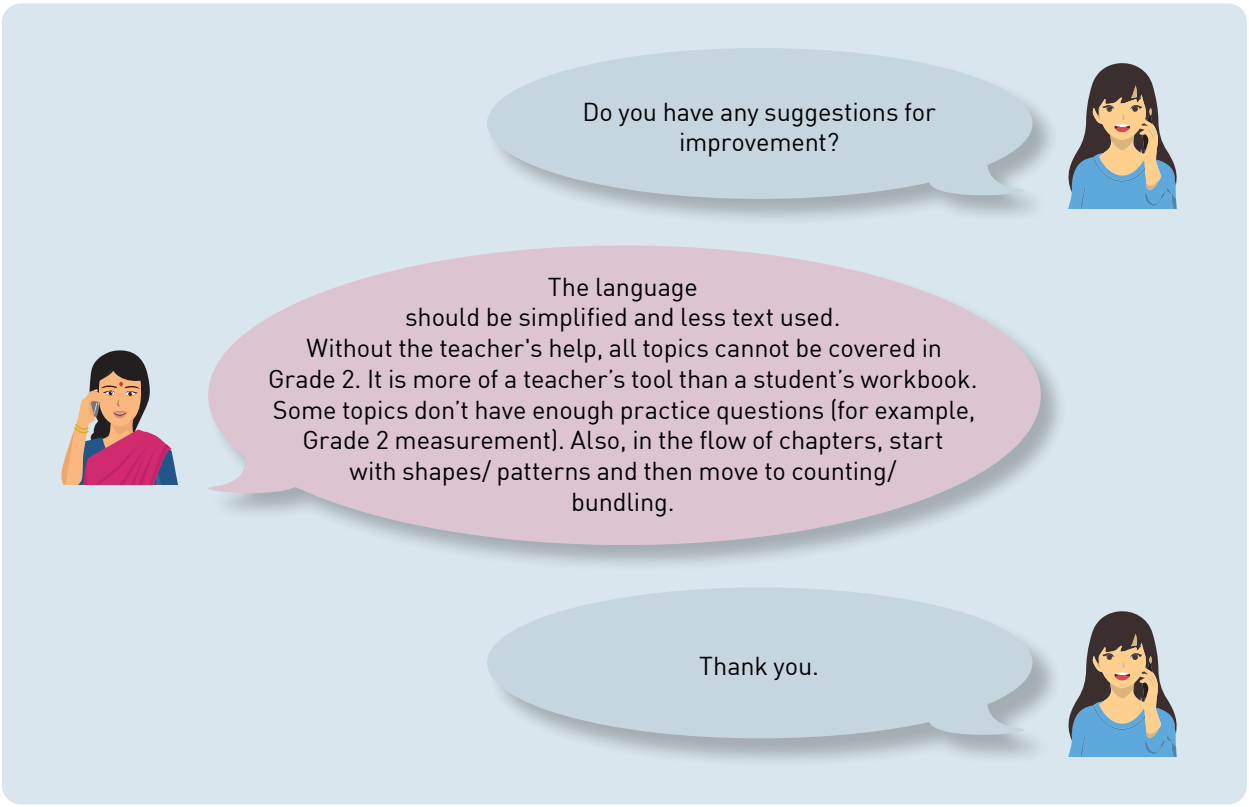
Can you share some examples of activities that children enjoyed?

Estimated length vs actuals is an activity that children love doing. They enjoyed filling out the table on page 79 in Chapter 7 of the Grade 1 textbook. It was fun to compare their guesses with their actual findings when they measured the suggested distances.

In Grade 1, Chapter 7, Page 82, "Water is very useful"- we have discussions on the usage of water. For example, how much water is needed for brushing? How many jugs of water are needed to fill a bucket? If it is glasses, more are needed- this is something they are able to say easily through the activity.



In Grade 2, Chapter 5, Playing with Lines, the introduction of dance movements is very interesting.



Here's another teacher in the survey who talks about the language difficulty:

> *"The linguistic aspects of teaching mathematics can be improved by giving specific vocabulary, phrases and sentence constructions that would help students develop practice of them consistently. There are many words which are not understandable across India (diyas, nimbu pani, bindis) and could be avoided especially in the English version of the book."*
>
> - Anagh, Azim Premji School, Bengaluru, Karnataka

Documented report of Kshama Chakravarthy's conversation with Bharathi. G. S and N. Pushpalatha, Teachers at Karnataka Public School, Sarakki, Bengaluru, Teach grades 1 and 2

Kshama Chakravarthy: What are your thoughts about the new version of the text books?

Bharathi: We commend the effort gone into the making of the new version. However we feel that in the Karnataka government schools scenario, where most students come into Grade 1 with no prior exposure to language or learning, these textbooks become very difficult for the teacher to use. Teachers have a lot more work to do and a lot of hand holding is required for students.

K: Hmmm.. So, is the language and vocabulary difficult?

Pushpalatha: Yes, the text should be much less and illustrations self explanatory wherever possible.

K: Okay. What do you think about the way the topics are covered?

B: The way the concepts are introduced and taught is very nice, where children learn a lot without actually realising that they are learning.

K: Can you explain this further?

B: By this we mean that the concepts are taught through examples, stories, TLMs etc. so students don't feel overwhelmed or fixated on the fact that they are learning something new or difficult. Grouping in tens and ones are introduced by different means.

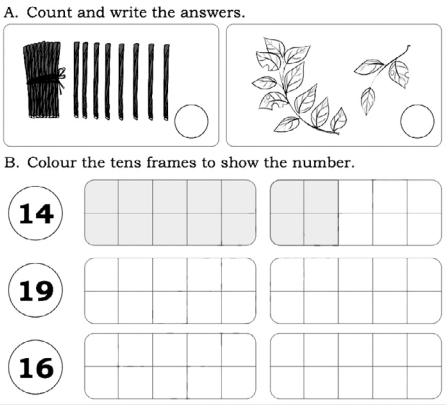


Figure 9: Grade 1, Chapter 4, Page 44

K: Do you find anything needing attention?

P: There is an error in the book that needs to be corrected. You will notice that based on the entries made in the third row, the fourth row cannot be filled with the numbers that they have printed. (See Figure 10)

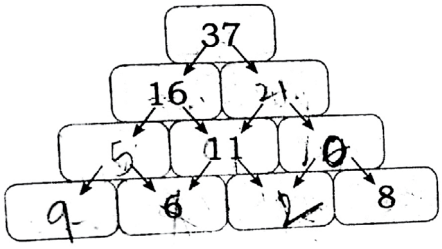


Figure 10: Grade 2, Chapter 6, Page 90

P: Also, the government schools are to use the bilingual books (English and Kannada), and the problems appear in both languages with the working/ solution to be provided each time, which is time consuming and irritating for the student. The suggestion is to ask the question in both languages and provide one single answer box/ working steps. (See Figure 11)

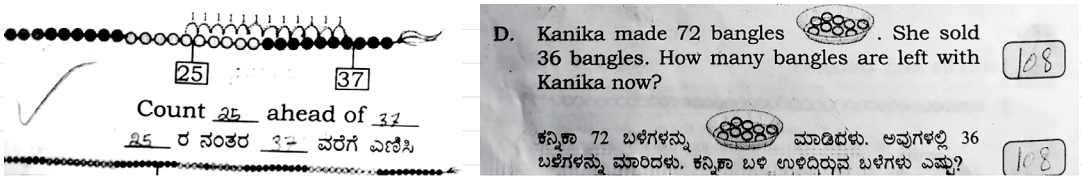


Figure 11: Grade 2, Chapter 6, Page 70 and 88 respectively

K: How about asking students to answer it just once?

P: We try 😊 They don't like leaving an empty box so they end up writing it in all of them, wasting time in the class. A change within the book makes it easier.

Note: The issue of bilingual textbooks falls under the purview of the state and not the NCERT. For the sake of preserving the conversation it has been retained here.

Having read snippets from different teachers about their experiences, let us now look at a detailed report shared by a teacher from APS, Uttarakhand, Ms. Sonia Kundu.

The Positives of the New Textbook (Grade 1)

Key Features of New Mathematics Textbooks

The revamped textbook aims to create a holistic and engaging learning experience for students by integrating innovative methods, interactive tools, and real-world applications. These features encourage curiosity, promote active learning, and develop a deeper understanding of mathematical concepts. Here are a few things that stand out for me in the new version.

1. Concept's introduction

Concepts have been introduced in simple ways, with visuals and game ideas making it easy for the child to understand. Here are a few examples.

Perceptual subitizing: It is intuitive. We can look at a small group of objects and instantly know how many there are without having to count. Grasping that numbers are made up of tens and ones is a foundational concept, paving the way for the understanding of larger numbers. Having a sense of "ten" as a group is essential for developing place-value understanding and performing mental calculations. This is dealt with well in the textbook.

Figure 12: Students are making number bonds by using Tens frame

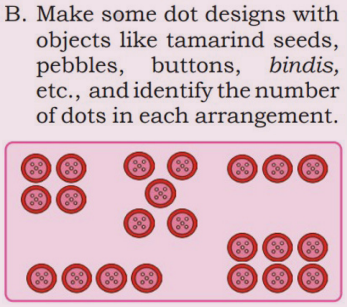


Figure 13: Grade 1, Chapter 4, Page 33

Conceptual subitizing: It involves recognizing a set of objects as made up of smaller groups. Take the example of dice: when a 6 appears, we might see it as two groups of three, which we understand to make six.

Numbers can be grouped and perceived in various ways. For instance, the number 8 can be thought of as 3 and 5 combined or as 4 and 4. Building skills in conceptual subitizing is crucial as it lays the foundation for many other mathematical concepts, such as composing and decomposing numbers, relationships between numbers and so on.

Use of Tens Frames: Tens frames are simple powerful tools that help students visualize numbers and understand place value.

The textbooks use tens frames for activities such as grouping, addition, subtraction, and identifying number patterns and provide a structured way to represent numbers. Tens frames prepare students for understanding place value by showing how numbers can be grouped in tens and ones. For example, seeing "13" as a full ten-frame (10) and three additional counters supports the idea of "1 ten and 3 ones", thus promoting a deeper understanding of place value and number sense. It encourages grouping and "making tens" for easier mental calculations (Figures 9 and 12).

2. Interesting Facts to Surprise Children

To spark curiosity and make learning enjoyable, the textbooks incorporate interesting and surprising facts which fosters a connection between mathematics and the real world.

Examples in the textbooks: Discussion about Sun temple, world's highest statue etc.

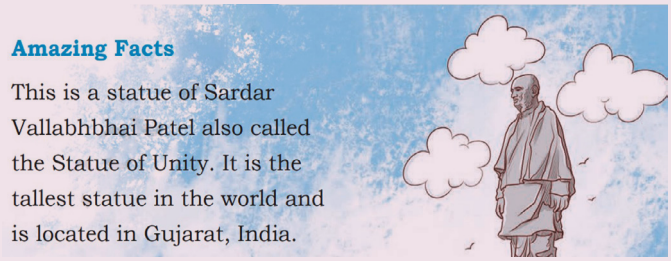


Figure 14: Grade 1, Chapter 7, Page 74

3. Interactive Pictorial Expressions

Visual aids and pictorial representations help make abstract concepts more concrete and relatable, by providing simple analogies. For example, colourful images, diagrams, and infographics for concepts like symmetry, shapes, and measurements, comic-style explanations for word problems and reasoning questions.

4. Inclusion of Suggestive Activities

Hands-on interactive activities are thoughtfully incorporated throughout the chapters to encourage active participation and provide opportunity for trial and error.

Examples in the Textbooks

- Using dice for addition or subtraction games
- Counting and grouping with objects like beads or sticks
- Drawing number lines and solving puzzles based on them

5. Well-Designed Flow of Concepts

The concepts in the new textbooks are structured in a logical manner, topics are introduced gradually, from simple to advanced/ complex ones. Chapters are interlinked, enabling students to build on previously learned ideas and see connections between different mathematical concepts, fostering a deeper and more integrated understanding.

- Prevents cognitive overload by introducing concepts incrementally.
- Reinforces prior knowledge while building new skills.

6. Reasoning Questions

The inclusion of reasoning-based questions encourages students to think critically and justify their answers.

- Enhances logical reasoning and analytical thinking.
- Develops deeper conceptual understanding rather than rote learning.

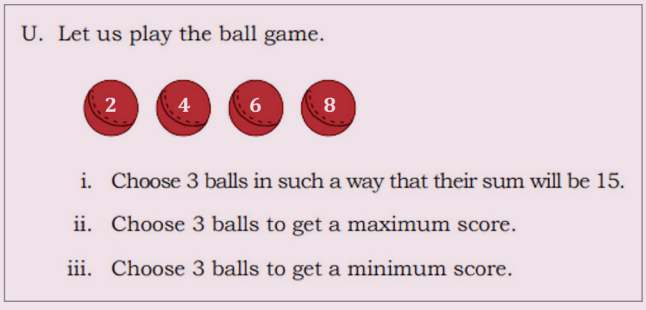


Figure 15: Grade 1, Chapter 13, Page 128

7. Games that promote Learning

Math-based games make math enjoyable and help reduce any fear or anxiety about the subject. They also encourage students to interact with their classmates, making learning a shared experience. It builds confidence and strengthens their understanding of key ideas. For example: Board games for arithmetic practice, puzzles for critical thinking, interactive group games for fostering collaboration.

Specific examples

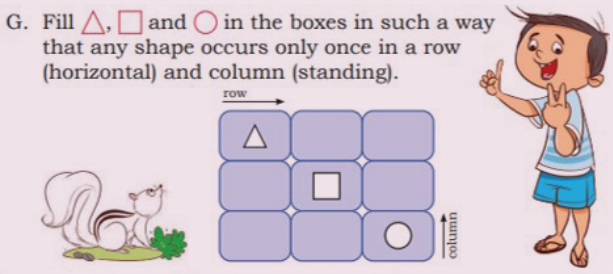
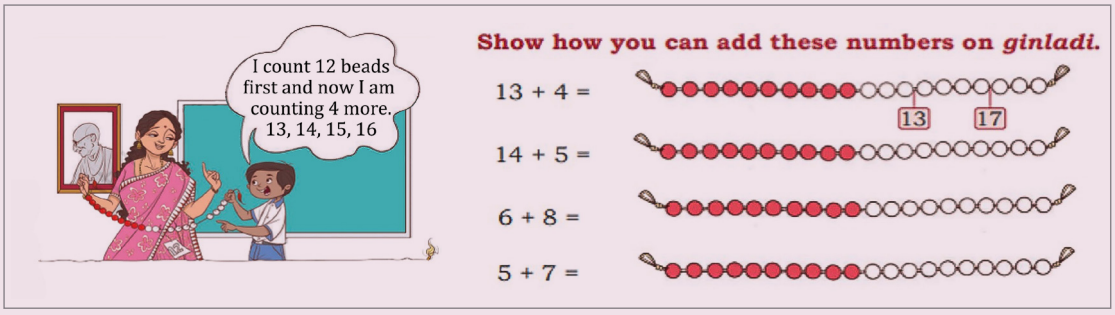


Figure 16: Grade 1, Chapter 13, Page 124

Textbook based activities

This page introduces addition and subtraction using visual tools like beads and number strips to make concepts engaging and relatable. Addition is shown through counting forward with beads, where children visualize sums by adding beads on a "ginladi" (e.g., 13 + 4 = 17). Subtraction is demonstrated as hopping backward on a number strip, illustrating how numbers decrease by taking steps back (e.g., 9 – 3 = 6). These activities provide a hands-on approach, reinforcing the concepts of sequential counting, addition, and subtraction, while transitioning students from concrete to abstract understanding.



Hop backwards on the number strip.

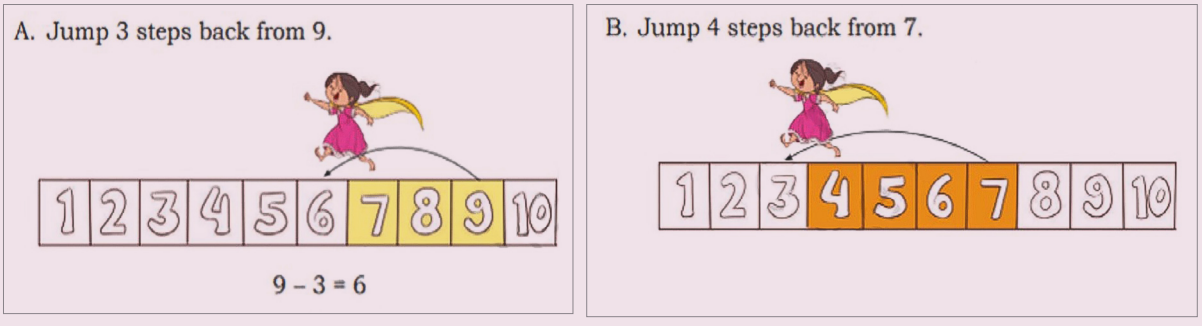
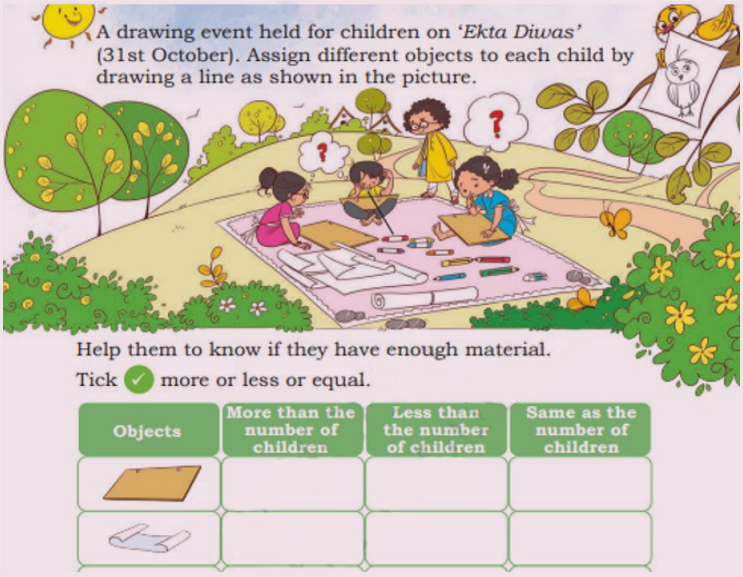
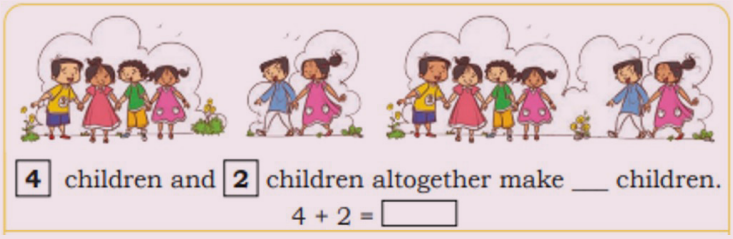


Figure 17: Grade 1, Chapter 5, Page 63 and Chapter 6, Page 67

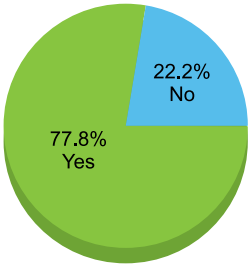
Suggestions for improvement

Current Issue	Suggested Improvement
The instructions lack clarity for situations when a number is rolled but all corresponding boxes have already been coloured.  Let us Play D. Play with your friend. Roll the dice and colour a box with the same number of dots as on the dice. Take turns with your friend and roll again. Figure 18: Grade 1, Chapter 4, Page 34	Add this line: *"If you roll a number and all the corresponding boxes are already coloured, skip your turn."* This ensures a smoother activity flow without confusion.
The limited space in the provided picture results in overlapping lines, making it difficult to understand the concept.  A drawing event held for children on *'Ekta Diwas'* (31st October). Assign different objects to each child by drawing a line as shown in the picture. Help them to know if they have enough material. Tick ✓ more or less or equal. Objects / More than the number of children / Less than the number of children / Same as the number of children Figure 19: Grade 1, Chapter 3, Page 23	1. Expand the visual area to avoid overlapping lines. 2. Alternatively, ask students to write the number of objects next to the picture and compare them to the number of children. This simplifies the process and introduces concepts like "more than," "less than," or "equal to.
Students tend to focus on counting all the images at once instead of understanding the relationship between the two groups, leading to errors.  4 children and 2 children altogether make ___ children. 4 + 2 = ☐ Figure 20: Grade 1, Chapter 5, Page 49	1. Encourage breaking the activity into steps, such as identifying and counting the two groups separately before summing them up. 2. Use guiding questions like: *"How many are in the first group? How many in the second? What happens when we combine them?"* This scaffolding can help students build their number sense and avoid mistakes.

Let us now look at the details from the survey that was taken by 90 teachers across the country, which gives us a flavour of how teachers perceive the new version of the Math textbooks of grades 1 and 2.

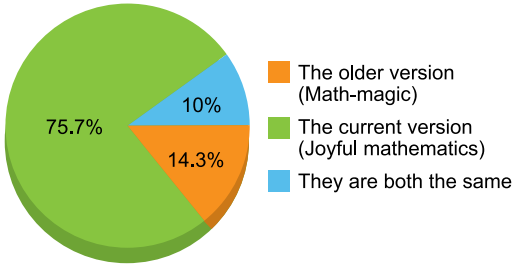
Have you taught students of Grade 1 or 2 from the old NCERT Math textbook?

90 responses



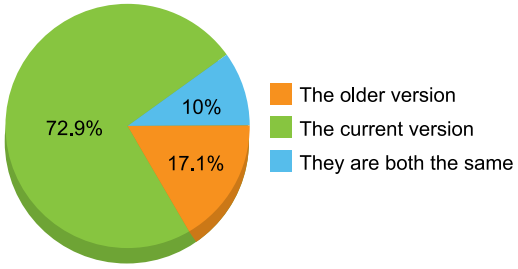
Which version of the Math textbook is more visual (more pictures)?

70 responses



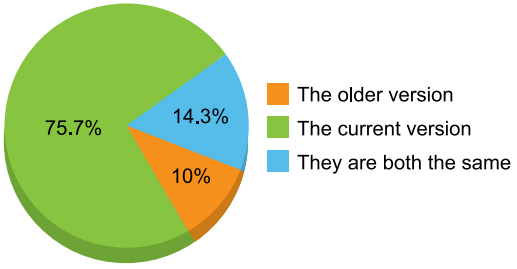
Which version of the Math textbook is more engaging?

70 responses



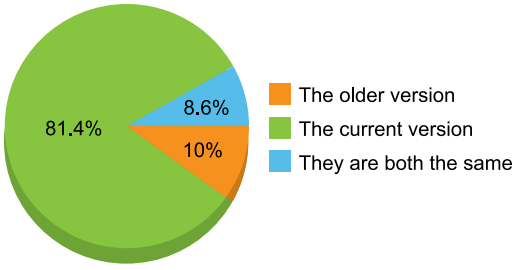
Which version has more instructions for the teacher?

70 responses



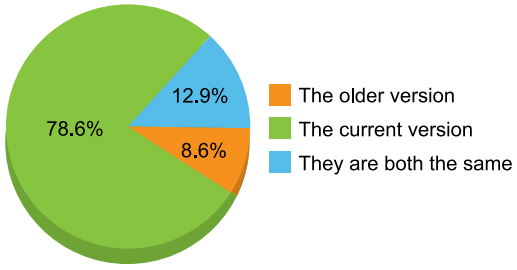
In which version do you think students enjoyed more?

70 responses



Which version do you prefer?

70 responses



Top 3 good things in the new version:

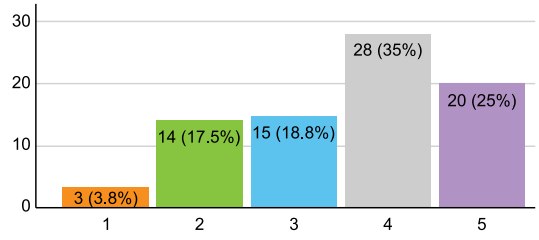
- A lot of pictures
- Good visualisation of many concepts
- Stories and activities

Top 3 things to change/ do differently in the new version:

- Increase practice
- Use words/ vocabulary that are familiar to all
- Increase the number of stories/ games/ activities

On a rating of 1 to 5, how much would you rate the current Grade 1 Math textbook?

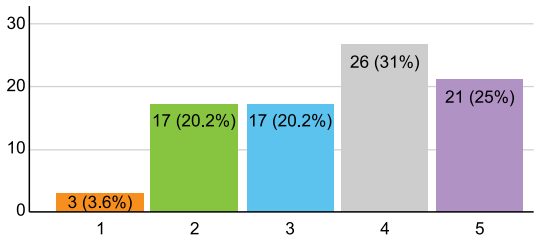
80 responses



60% have given a rating of 4 or above for the new version of the Grade 1 textbook

On a rating of 1 to 5, how much would you rate the current Grade 2 Math textbook?

84 responses



56% have given a rating of 4 or above for the new version of the Grade 2 textbook

The new NCERT Math textbooks for Grade 1 and 2 have been well-received by teachers who have praised the thoughtful design and approach embedded in the chapters. From easy access to additional materials through simple QR codes, to clear instructions, the teachers feel that the textbooks offer a range of engaging learning methods, including stories, rhymes, poems, pictures, real-life activities, cultural references, and project work within each chapter. Additionally, the inclusion of puzzles has sparked excitement among both students and teachers alike.

However, there are some suggestions for improvement, particularly regarding the flow of the chapters and the amount of text in Grade 2, the space allocated for practice exercises, the number of practice questions, as well as the need for clearer guidance for teachers on the significance of certain concepts and different methods for teaching them. These areas could be explored further in the next iteration of the textbooks.

In conclusion, the new version of the textbooks is a promising step forward, fostering a more interactive, hands-on, and holistic learning experience. While there are areas for refinement, the positive feedback from teachers and the innovative features of the textbooks suggest that they are on the right track towards making mathematics a more joyful and meaningful experience for young learners.

References

1. Joyful Mathematics: Class 1 (2023), NCERT https://ncert.nic.in/textbook.php?aejm1=0-13
2. Joyful Mathematics: Class 2 (2023), NCERT https://ncert.nic.in/textbook.php?bejm1=0-11
3. The National Curriculum Framework for Foundational Stage (NCF-FS) (2023)

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Understanding a Circle through Paper Folding

Rahul Singh Rathore

How can a paper folding activity help to approximate and understand a circle? Read on to find out the author's experience with Class 4 students.

In Class 4, I was working towards helping the learner acquire understanding about shapes around her/him. In particular, I wanted my students to identify the centre, radius and diameter of a circle.

In order to check their pre-requisite understanding and then proceed towards my goal, I made the following plan for the students:

- Observe and identify round and circular shapes from the surroundings.
- Collect objects that make a circular trace.
- Make circles using coins, bangles, etc.
- Make a circle using paper folding.
- Understand increasing and decreasing circles using threads of different lengths.
- Learn and use vocabulary associated with the circle.
- Draw a circle using a compass.

The children could easily identify round and circular shapes and used bottle caps, bowls, bangles, protractors, etc., to trace out circles. I then asked them what they would do if they wanted to make smaller or bigger circles without using such objects. After some thought, they tried freehand sketches (see photo), but they were able to see that these were not as perfect.

Showing students sketching freehand circles.

Keywords: Geometry, Circle, Conceptual Understanding, Kirigami, Experiential Learning

Making a Circle Using Paper Folding

When I talked about making a circle with paper in the classroom, all the children were curious about how this circle would be made.

Paper folding and cutting using scissors, also known as kirigami, is a fascinating practice that merges creativity with geometry. While most people associate paper craft with creating shapes such as cranes or flowers, the process of creating perfect geometric shapes, such as a circle, can also be initiated through this simple yet profound craft. Folding a circle from a flat piece of paper isn't as intuitive as making a square or triangle, but it is certainly achievable with the right techniques.

This article explores how paper folding can be used to create a circle, blending artistry and mathematics in a way that is both enjoyable and educational.

The Challenge of Creating a Circle with Paper Folding

At first glance, creating a perfect circle out of paper folding may seem impossible. A circle, after all, is defined as a set of points that are equidistant from a central point. In paper, where each fold tends to create straight lines or sharp angles, achieving a true curve can seem like a contradiction.

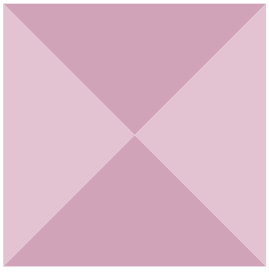
However, it is possible to approximate a circle very closely using the principles of kirigami. The process typically involves a series of folds that help refine the shape of the paper until it resembles a circular form.

Basic Techniques for Creating a Circle Through Folding

There are multiple methods to create a circle through folding, but all involve precise folding and symmetry. Here is a general step-by-step approach to creating a circle. Begin with a square piece of paper. While it is possible to start with rectangular paper, a square provides symmetry and makes the folding process easier to follow.

Step 1. Fold Diagonally

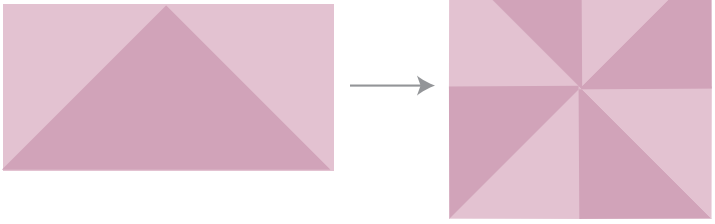
To start, fold the paper first along one diagonal (from one corner to the opposite corner) and then along the other. When we did this and then opened the paper, we found a cross in the centre of the page.



Step 1

Step 2. Fold in Half Horizontally and Vertically

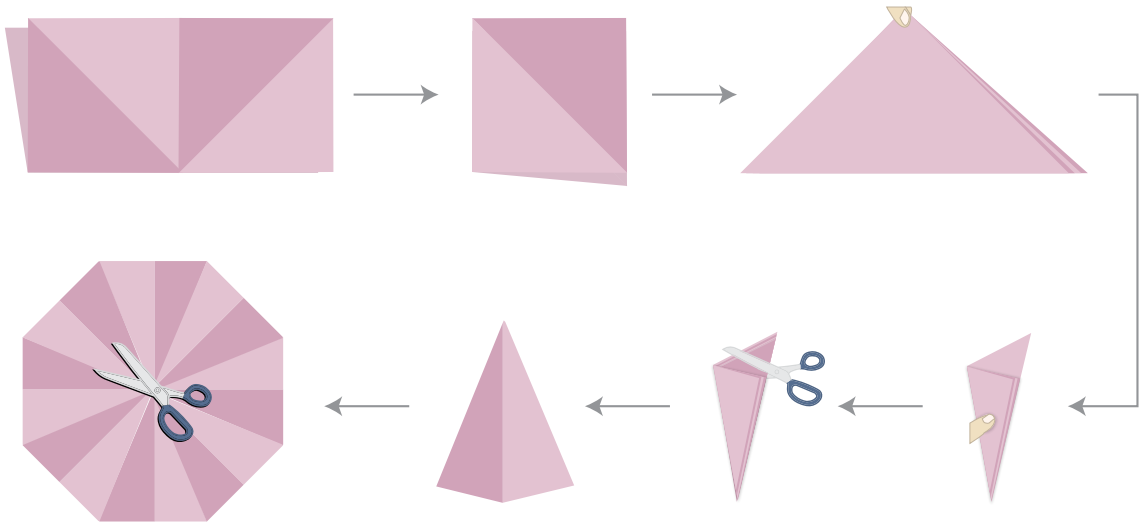
After this, fold the paper in half both horizontally and vertically. When we opened the paper, we saw that two more lines had formed in the middle which were passing through the cross that we had observed earlier.



Step 2a Step 2b

Step 3. Fold the Corners to the Centre

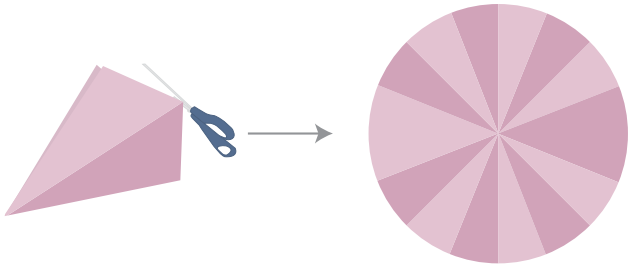
After this, fold the paper into a rectangle and then into a square. Bring the two opposite corners of the square together to form a triangle. Join the two matching sides of the triangle together to make a smaller triangle and cut the page with extra protruding from the top. Open the page out, you will see a polygon.



Step 3

Step 4. Refine the Shape

After seeing the polygon, fold the paper again and eliminate the sharp points, gradually creating a smoother, more circular appearance. While these folds won't create a perfect mathematical circle, they help approximate one.



Step 4

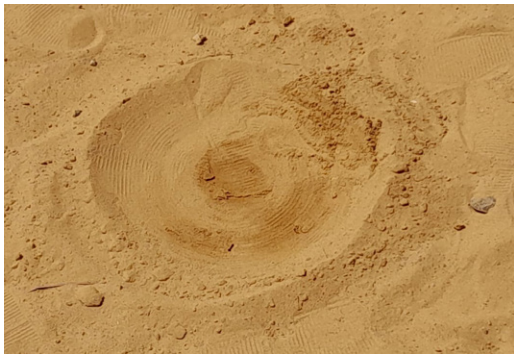
Why Paper Folding Can Approximate a Circle

The reason that a paper fold can approximate a circle lies in the properties of geometric approximation. As you fold the edges toward the centre, you are progressively reducing the number of straight edges and forming a shape closer to the ideal circle. With each fold, the paper moves toward a more "curved" geometry, even though the result remains an approximation. After we did this, the children coloured in each radius section.

What was important to note was that at this point, the children said that in each coloured part, there is a straight line coming from the centre to the edges, whose length is the same from all sides, then we discussed the term ‘radius’ and I explained that this was any line from the centre to the edge of the circle. All such lines were of the same length- something that the children could

easily see with the folding they had done. And the children also noticed that there is a fold from one edge to the other passing through the centre of the circle, which is double of the radius, and I explained that this is called the diameter of the circle.

After the fold lines were used to discuss centre, radius and diameter of a circle, I asked the children to go out of the classroom and think of ways to draw circles using what they had just learnt.



Children make a circle using their feet

Children used rope tied to sharp objects on both sides

In both cases, what made me happy was they recognised the significance of the centre and of the radius. In fact, by changing the length of the rope, they were able to make bigger and smaller circles. After this, there was a discussion about how to make a circle in their notebooks, then one of the children said that he had seen some children making circles with a compass. Using the experience with the rope, the following method was thought out.

Using a Compass

1. Fix a pencil in the compass slot and place the compass point where you want to mark the centre of the circle.
2. Adjust the compass to the desired radius.
3. Keeping the point fixed, rotate the compass to draw the circle.

Arriving at definitions

- Centre: The central point of the circle.
- Radius: The distance from the centre to any point on the circle.
- Diameter: The distance across the circle, passing through its centre. (Twice the radius)
- Circumference: The distance around the circle.
- Arc: A part of the circle's circumference.
- Chord: A line segment connecting two points on the circle.
- Tangent: A line that touches the circle at exactly one point.

Activities and Questions to Learn About Circles

1. Draw a Circle: Use a compass or a round object to draw a circle.
2. Measure the Radius and Diameter: Use a ruler to measure the radius and diameter of a circle. This was done in the classroom.
3. Walk Around a Circle: Walk around a circular object, like a frisbee or a plate, to understand the circumference.

4. Discuss the characteristics of a circle. These were the conclusions:
    - A circle has no corners or edges.
    - All points on the circle are equidistant from its centre.
    - You can draw a circle by connecting all points that are the same distance from a centre point.

Circles are all around us! Understanding circles can help you in math, science, and even in our daily life. Remember, a circle is a round shape with a centre, radius, diameter, and circumference.

Applications and Insights

Creating a circle through paper folding is more than just a craft; it also offers valuable insights into geometry and symmetry. By folding paper in this way, you can gain a deeper understanding of how shapes relate to one another and how straight lines can be manipulated to approximate curves. This technique also demonstrates how paper can be used to explore mathematical attributes such as precision, approximation, and transformation.

By engaging in this hands-on process, you not only develop a greater appreciation for the beauty of geometric shapes but also experience first-hand how the laws of geometry can manifest in the real world.

In pedagogy, these kinds of paper folding activities can be used to teach students about circles, symmetry, and approximation. Through visual and tactile learning, students can see the relationship between flat, straight shapes and curved, circular forms.

Conclusion

While paper folding cannot create a perfect mathematical circle, it provides a fascinating way to approximate this elegant shape using simple tools and methods. The process of folding paper teaches us about the interplay between straight lines and curves and offers a tactile understanding of mathematical principles. Whether for artistic expression, educational purposes, or personal curiosity, the act of creating a circle through paper folding offers a unique and rewarding exploration of geometry.

Reference

1. Learning outcome - Identify the centre, radius and diameter of a circle from Sims and NCF 2005.
2. https://youtu.be/cSst1EW7LtY?si=X37BdCikTrRj6EwO

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Crafting Questions for Diverse Learning Goals

Shekh Mohammed Zahid

Many children find mathematics tests *'scary'* because they perceive them as high-stake assessments of their capacity to handle complex methods and formulas. Usually, tests and exams include questions designed to assess a student's understanding of a specific topic. But are questions only for testing and evaluation? A closer look reveals that questions have the potential to accomplish much more. For example, Socrates used questions to explore complex ideas, challenge assumptions, and pursue wisdom.

Typically, the questions we use in exams are straightforward and lack thought-provoking elements. Children are naturally curious thinkers, and a direct mathematical question doesn't always spark their curiosity. Effectively designed questions with specific purposes, such as promoting appreciation of concepts, developing skills, and understanding new properties, can significantly benefit students. These types of questions should be part of our assessments in order to enhance the student's deep understanding of concepts and to shift away from rote learning methods.

Consider the two questions given in Figure 1 and Figure 2. Figure 1 is a standard question for selecting the correct time. Although it is a good diagnostic question, it does not involve any thinking elements. The question is slightly modified by removing the minute hand from the clock, as shown in Figure 2. This question encourages thinking and teaches us how to estimate time.

What is the time on the given clock?

A) 7:03 B) 7:30 C) 3:07 (D) 7:15

Figure 1

The minute hand of the clock is broken and only the hour hand is working, as shown. What time does the clock show?
(Choose the most suitable option.)

A) 4:00 B) 4:10 C) 4:30 D) 4:50

Figure 2

Keywords: Assessment, design, learning objectives, thinking skills.

Most of the tests conducted nowadays are based on memorising facts rather than understanding concepts. With the introduction of the National Education Policy (NEP) in 2020, new initiatives are being taken to reform education systems and shift the objective of assessment towards higher-order thinking with PARAKH (Performance Assessment, Review, and Analysis of Knowledge for Holistic Development). PARAKH aims to assess a student's overall development, taking into account not just their academic performance but also their cognitive, social, and emotional growth.

At Open Door Education (www.opendoor education.in), we create questions that not only assess understanding but also inspire students to think creatively when solving problems. We design a variety of questions that appreciate concepts and help students to develop a deeper understanding of them. In this article, we showcase some of our questions to demonstrate how thoughtfully designed questions with various objectives can spark discussion and how a range of questions on a specific topic can be developed to assess mastery.

Designing Questions That Encourage Thinking

The question shown in Figure 3 is designed to indirectly teach students that "the set of whole numbers has no largest element." When students attempt this problem, they typically begin by writing down the largest number they can think of. However, they soon realize that they can always choose a number larger than the one they just wrote, leading them to understand that there is no largest whole number. This question demonstrates how presenting a mathematical problem in a different manner can transform it into a more interactive and engaging activity.

If you consider all the whole numbers, which of the following can be written on a piece of paper?

I. The largest whole number.

II. The smallest whole number.

A) Only I B) Only II

C) Both I and II D) None

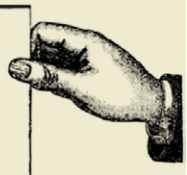


Figure 3

In mathematics, there are two important numbers: 0 and 1. 0 is defined as the additive identity for whole numbers, and 1 is known as the multiplicative identity for whole numbers. It will be interesting to understand the significance of these technical terms by solving a question. We have designed a question (Figure 4) for Class 6 students that highlights the application of these mathematical terms.

The product of some Chinese numbers is shown. If the Chinese numbers have all the properties of whole numbers, then which of the following numbers does the symbol 壹 denote? *(Hint: Recall the properties of whole numbers)*

壹 × 柒 = 柒 | 壹 × 玖) = 玖)

A) 0 B) 1

C) Can be either 0 or 1 D) Cannot say

Figure 4

If x and y are any whole numbers then $x + 0 = x$ and $y \times 1 = y$. We see adding 0 to any whole number will result in the same number and multiplying 1 with any whole number will result in the same number. Looking at the bigger picture, we can conclude that this property of numbers applies to any numerical system. To solve the question in Figure 4, students just need to observe the patterns in these symbols and know that 1 is the multiplicative identity.

Now, let's explore how we can use a question to help students discover some interesting properties.

We know that a number is divisible by 3 if the sum of its digits is divisible by 3. A simple observation is that even if we shuffle the digits and form a new number, the number will still be divisible by 3 because the sum of the digits will remain the same. The question in Figure 5 is posted for 5th graders in some schools. Although the question is simple, it encourages students to explore and discover fascinating mathematical properties. For example, the question in Figure 6 presents a slightly more challenging and modified version of the question for the students.

There are four different digits inside the bag. Fill the boxes using these digits and make 4-digit numbers. *(Note: No repetition of digits is allowed)*

Which of the following is correct about the number formed?

A) All the numbers formed will be divisible by 3.

B) Some of the numbers formed will be divisible by 3 but some will not be.

C) None of the numbers formed will be divisible by 3.

Figure 5

What interesting properties do you observe when solving the question in Figure 6?

John fills the same digits in all the boxes. Which of the following options is correct?

A) All the different numbers formed will be divisible by 12

B) All the different numbers formed will be divisible by 2

C) All the different numbers formed will be divisible by 3

D) Cannot conclude any of these options

Figure 6

Fraction Problems with Varying Difficulties for Primary Students

To evaluate mastery of a topic, it is essential to use a variety of questions with different difficulty levels. Developing multiple questions for a single concept can be challenging. Additionally, each question should incorporate elements that potentially identify some misconceptions or difficulties. By diversifying how questions are presented and adding new visuals, we can develop a wide range of questions on a specific topic, see Figure 7. Questions 1-10 are designed around fractions and have different difficulty levels. The questions include multiple-choice (MCQ) and interactive drag-and-drop types.

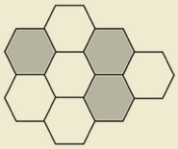
Each of these questions has a specific objective along with some distractors. Q1 focuses on whether students understand how to represent fractions. Q2 tests the misconception that to represent a fraction in the given figures, the shaded parts must be equal.

Some students might be confused about how to represent a full or whole part when dealing with fractions. Q3 aims to determine whether students understand how to represent a whole (or "full") in the context of fractions.

Although Q4 is fairly straightforward, many students hold misconceptions about representing fractions in the context of objects. In this question, students might mistakenly select 3/4 as the fraction representing blue pens, assuming they should place the number of blue pens in the numerator and the red pens in the denominator.

Interestingly, dividing a symmetrical shape in half is easier than dividing an asymmetrical one. Q5 is an interactive question designed to illustrate this concept. Q6 is a misconception question that tests whether students understand how to correctly represent the fraction 1/2. Students might choose the answer 'Yes' by thinking that the figure is divided into two parts; however, they usually miss the point that the divided parts must be equal.

Q1. Which fraction represents the shaded portion in the given figure?



A) $\frac{3}{5}$ B) $\frac{5}{3}$ C) $\frac{8}{3}$ D) $\frac{3}{8}$

Q2. Which of the figures shown has $\frac{1}{5}$ part shaded?

Figure I

Figure II

A) Only figure I B) Only figure II
C) Both figure I and figure II D) None

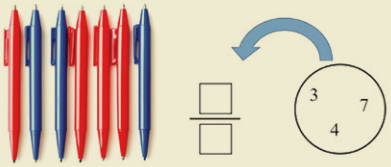
Q3. An apple is cut into six equal pieces. Which fraction represents the whole apple?

One whole apple

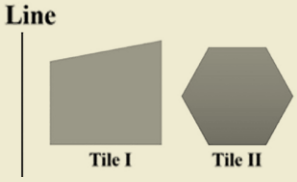
Six pieces of apple

A) $\frac{1}{6}$ B) $\frac{3}{6}$ C) $\frac{6}{1}$ D) $\frac{6}{6}$

Q4. There are 4 red pens and 3 blue pens. What fraction of the total number of pens is blue? *(Drag and drop the numbers from the circle to the blank boxes)*



Q5. Which of the following tiles can be easily broken into two equal pieces? *(Move, rotate and drop the given line to divide the tiles)*



A) Tile I B) Tile II

Q6. Is $\frac{1}{2}$ of the whole shape shaded in the figure?

A) Yes B) No

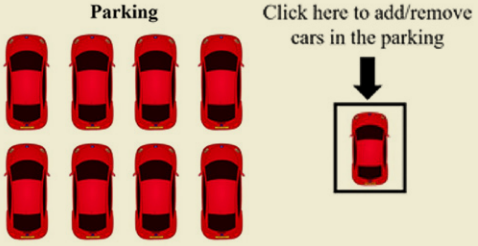
Q7. Who ate more dosa?

Ben eats $\frac{1}{2}$ of the above dosa

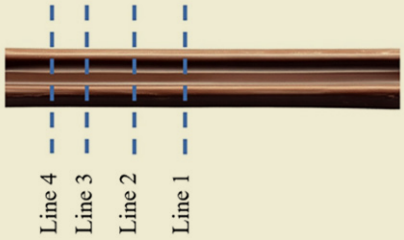
John eats $\frac{1}{2}$ of the above dosa

A) Ben B) John
C) Both Ben John ate an equal amount of dosa

Q8. The picture shows the parking lot after half of the cars have left. Initially, how many cars were parked there? *(Left-click on the tab to add a car, and right-click to remove a car.)*



Q9. Where should the initial cut be made to divide the chocolate into three equal parts?



A) Line 1 B) Line 2 C) Line 3 D) Line 4

Q10. Approximately _____ of the glass is full of water. *(Choose the best suitable option and fill in the above blank)*

A) $\frac{4}{8}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{1}{4}$

Figure 7

It is worth considering whether half of two similar objects of different sizes are equal. Q7 addresses the idea that, numerically, halves are equal; however, half of two similar objects of different sizes are not equal. Q8 is an interactive question that explores whether knowing half of a number allows us to determine the whole.

Most of the time, we are asked to calculate one-third of an object; however, in Q9, it becomes interesting to consider where the first cut should be made if we want exactly one-third of an object. Lastly, Q10 is a question on the estimation of fractions.

We observe the creation of various questions around a single concept, which not only assesses student's understanding of fractions but also encourages them to think more deeply when tackling specific problems.

Conclusion

Question design plays a crucial role in helping students achieve diverse learning objectives. There is significant potential to craft various questions to fulfil multiple learning purposes. The purpose of assessments should extend beyond testing student's knowledge to also include questions that challenge their thinking and spark curiosity.

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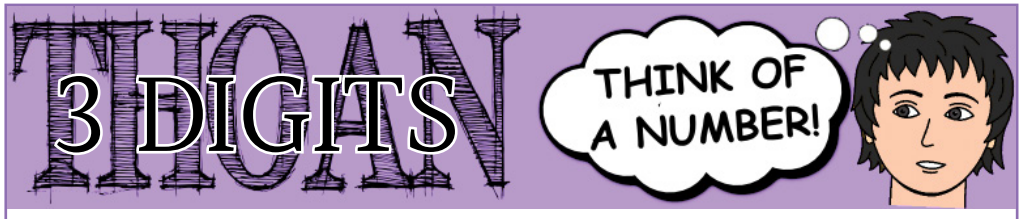


Think of a three-digit number 'abc'. Now, move the first digit 'a' to the end so that it becomes the last digit of the number, which would be 'bca'. Subtract the new number from the old.

For example, if the first number 'abc' is 356, then the new number 'bca' is 563.

I claim that if I know the difference and the first digit of the original number then I can guess that number. [If I know that the difference is -207 and the first digit is 3, I can guess that the first number was 356].

Can you guess how I do this? Try with some other 3-digit numbers and send in your solutions to AtRightAngles.editor@apu.edu.in

Look out for more THOAN activities from **Yathiraj Sharma** in upcoming issues of At Right Angles.

Exploring Patterns: Learning with Numbers in Grade 2

Jagrati Mehra

In Grade 2, students are introduced to the fascinating world of number patterns. These patterns help them develop important skills in recognizing sequences, understanding relationships between numbers, and boosting their mathematical skills. One engaging way to teach this is through exercises where students fill in blanks in grids using the numbers around the blanks to identify patterns. In this article, we'll explore how students can uncover these patterns, use tables, and develop their thinking skills through exercises involving numbers from 1 to 100.

Recognizing simple sequences

This activity involves students to use the patterns in the 10 × 10 number grid given in Figure 1.

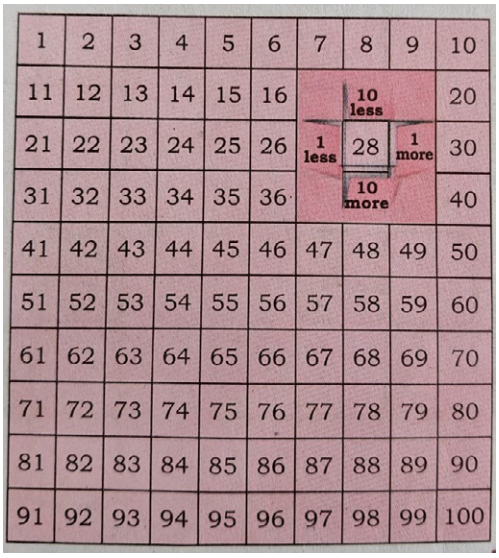


Figure 1: A 10 by 10 number grid

One of the easiest ways to get started with number patterns is through simple sequences, where students need to fill in the blanks. Here's an example from the above grid:

Example 1

	24	

The pattern is straightforward. The numbers go in a sequence with a difference of one unit: 23, 24, 25. So the filled-in sequence looks like this:

23	24	25

But what happens if the numbers go in a different direction? Or if we need to find numbers above or below the given ones?

Finding numbers above and below: building spatial thinking

So let's take this idea a step further and think about the numbers above and below a specific number in a table of numbers from 1 to 100.

Keywords: Numbers, patterns, sequences, connections, rules, communication

Example 2

Look at the number 14. What are the numbers directly above and below it?

Students can answer just by looking at the number chart. When I tried without showing them the number chart, they were still able to give the correct numbers 4 and 24, but the interesting part was that they were struggling to express the reason behind their answers.

Eventually, they came up with "4 is 10 less than 14" and "24 is 10 more than 14"

Once students repeat this with different numbers, they begin noticing that the table forms a grid of numbers with a predictable relationship between them. Spatial thinking is the ability to visualize and manipulate objects, numbers, or patterns in a structured space. In the context of identifying numbers above and below in a 1-to-100 grid, spatial thinking develops as students begin to mentally organize and interpret numerical relationships based on their positions in a structured arrangement. Understanding this spatial arrangement helps build a deeper comprehension of how numbers are related.

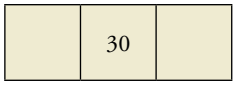
Exploring Sequences with Missing Numbers: More Complex Patterns

Given Example 3, students may fill in the next number as 21, but it's not as obvious as in the previous case because 20 is at the extreme end of the row with no numbers to the right of it. To really understand the sequence, students need to notice that the numbers are increasing by one each time, forming a simple pattern.

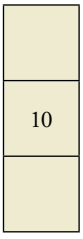
This kind of sequence encourages students to think about how numbers follow each other and how sequences can be extended. This makes them independent of the number chart.

What about sequences that involve a higher level of thinking? For example, when students are given the numbers

Example 4

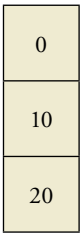


Example 5



They might begin to question what comes next. Often, students will try different strategies to fill in the blank, and here's where the beauty of spatial thinking shines through.

I noticed that some students kept it blank but others finally came up with



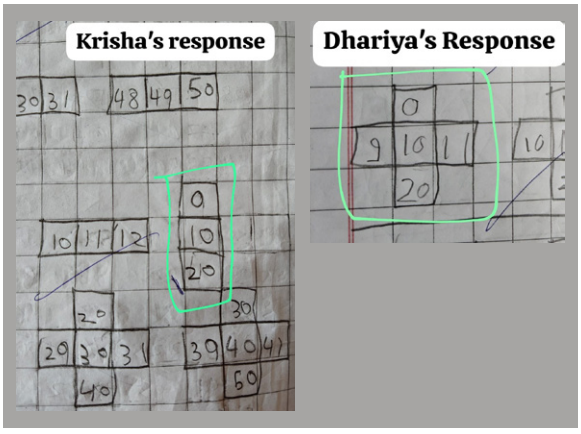


Figure 2a and 2b: Student responses

Turning the Table of Numbers from 1 to 100 into a Flexible Tool

Here are the puzzles which they worked on

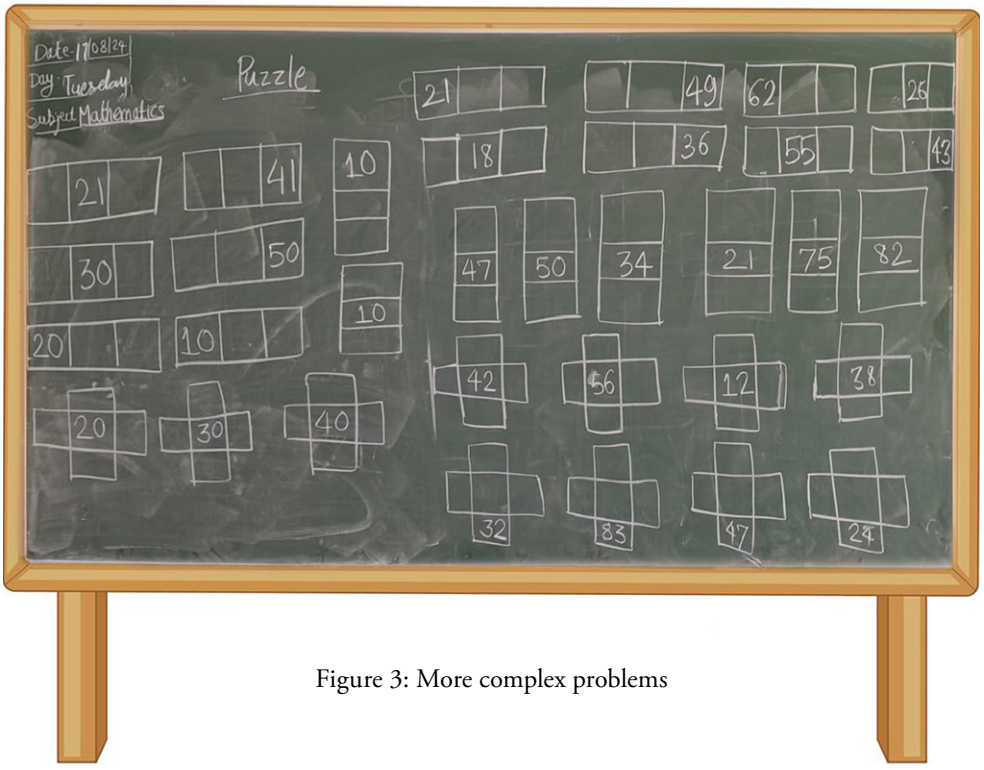


Figure 3: More complex problems

Introducing so many abstract questions on the board isn't something that happens on the first day. I had already introduced this topic earlier, starting with simpler concepts such as identifying the numbers that come before and after a given number. We then moved on to arranging these numbers in basic puzzles. Gradually, I increased the complexity of the puzzles, but we hadn't explicitly discussed the logic of "1 less or 1 more" or "10 less or 10 more."

I hadn't given such complex questions before. However, an observer encouraged me to try these examples on the board. To my surprise, many students were able to answer them, coming up with their own reasoning. Of course, some students were still puzzled, and a few responded with the argument that "there is no number nearby," which is a valid observation in its own right.

At first, the table of numbers from 1 to 100 may seem compact, and it's easy to just look at it as a fixed grid. But as students explore patterns, they begin to "bend" this grid to fit their needs. For example, when faced with the sequence 20, 21, 22, many students might say: "Wait, why does this table only show numbers in rows of ten?"

This shift in thinking — where the number table no longer feels rigid — is a crucial skill. By the end of these exercises, students understand that patterns aren't just confined to one way of looking at numbers. They can move and adjust them based on the context they're working in.

Scope of this material

- This activity helps students to understand different concepts such as increasing and decreasing order, preceding and succeeding number, etc.
- As students get more confident with vertical and horizontal patterns, we could include diagonal patterns which can lead to recognising the multiplication tables at a later stage.
- A suggested Higher Order Thinking Skills activity is for the teacher to change the number of columns in this grid to say,

20 columns (or later 15 columns) and get the students to see relationships between the numbers. They should be able to (eventually) see that to the left or right the numbers still decrease or increase by 1 respectively, while numbers decrease or increase by 20 (or 15) above or below respectively. Of course, diagonal patterns will be even more complex!

Developing Critical Math and Thinking Skills

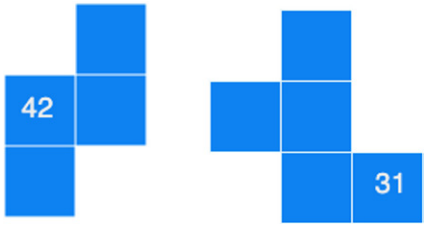
Number patterns are a foundational part of Grade 2 mathematics. As students begin to fill in blanks, find numbers above and below, and identify the next number in a sequence, they develop a stronger understanding of numbers and their relationships. These activities not only help them practice arithmetic but also foster important problem-solving skills and the ability to think spatially.

In the example of 20, 21, 22, students realize that numbers don't just belong to a rigid grid; they can expand and shift to form new sequences. This exploration of number patterns and spatial relationships helps students become more flexible in their thinking — an essential skill in math and everyday problem-solving.

As they continue to practice and explore, they will become more confident in their ability to identify and create patterns, setting the foundation for more advanced mathematical concepts in the years to come.

Editor's Note

The concepts presented in this article can be expanded to include more challenging problems and activities for children. Students could be introduced to triominoes, tetrominoes, and even pentominoes, which can be incorporated into engaging worksheets. For example, a worksheet could ask students to fill in blank spaces within a given pattern (as shown below) and provide reasoning for their answers.



Secondly, the activity in this article focuses only on addition and subtraction by 1 unit and 10 units. It could be enhanced by including addition and subtraction with other units, allowing children to practice skip counting and reverse skip counting.

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Montessori Approach: An Introduction to Selected Materials and How to re-create them (Part 2)

Kshama Chakravarthy

Ever wondered how simple, hands-on materials can unlock a child's potential? Discover cost-effective ways to bring Montessori magic into your home while reaping the benefits of a method that fosters independence, creativity, and a lifelong love of learning.

As the title suggests, this article is the second in the 2-part series that covers a total of six Montessori materials. The first part, in the November 2024 issue of At Right Angles, covered materials 1-4 i.e., pink tower, brown stairs, long rods and number rods [1]. This article covers cylinder blocks and knobless cylinders.

The Montessori Curriculum consists of five key areas of study: Practical Life, Sensorial, Mathematics, Language, and Culture. There is a set of Montessori materials for each area, focussing on a key knowledge or skill. In this article, we will look at some of the sensorial materials, specifically those that allow for visual discrimination (thin, thick, tall, short). As you read through, you will realise that there's so much more to them- mathematics, language, attention to detail and a lot more that is intertwined! It will hopefully give a glimpse of what can be done in the class and what to expect as an outcome from it.

Material 5: Cylinder blocks

There are 4 sets of cylinder blocks, consisting of 10 cylinders each, placed in a case with sockets to hold each of them. In one set, the cylinders are placed from big to small (Figure 1- bottom). In another, they are placed from thick to thin (Figure 1- left side). In the third they are placed from thick and short to thin and tall (Figure 1- top), and in the fourth set they are placed from tall to short (Figure 1- right side).

Figure 1

Keywords: Montessori, TLMs, low-cost materials, DIY

Figure 2: Holding the cylinder by the pin is preparing them to hold a pencil eventually

How to use: The sets of blocks are introduced to children one by one, in the order mentioned above. The child places the block on the mat and removes all the cylinders. The cylinders are placed randomly and the child is asked to put them back in, in the right place. Notice that the activity cannot end with cylinders wrongly placed or some holders left empty. A child will be able to, by trial and error initially, place all the cylinders correctly before moving to the next set. This is one example of how Montessori materials are self-corrective.

A subtle point to note is that activities such as this, help prepare the child towards holding a pencil and developing fine motor skills. Seemingly small things such as this are paid attention to in the creation of each material. (Figure 2)

Additional Activities

Activity 1: Once the child is comfortable placing the cylinders correctly, the blocks are removed and the cylinders are placed randomly on the mat. The child is now asked to arrange them in increasing or decreasing order, without the block with sockets available to confirm their placement (one block set at a time).

Activity 2: Two block sets are placed in an L-shaped arrangement (big to small and thick to thin for instance) and all the cylinders in them are removed and mixed up. The child is now asked to place each of them in their right place. This activity is repeated with 3 sets and eventually with all 4 sets (this may be more apt for 6 year-olds).

Activity 3: The cylinders are placed in a tray and kept in another room. An indicator (or one could use a pencil in its absence) is kept in one of the sockets. The student is asked to go to the tray and pick the right cylinder that can be placed where the indicator is kept. This activity involves visual recognition of the dimension of the cylinder, keeping it in memory and identifying the right cylinder from the tray. Multiple attempts are allowed initially in case the child needs them.

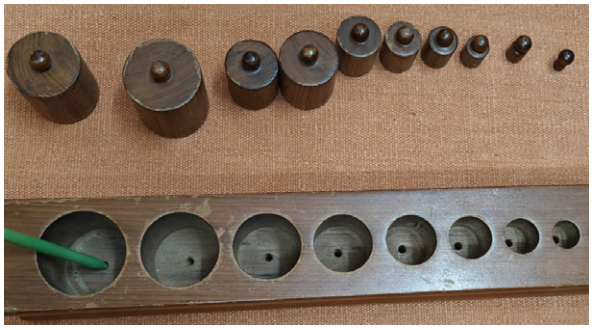

Figure 3: Activity 3

Material 6: Knobless cylinders

Another variation of the cylinder blocks is the knobless cylinder (or coloured cylinders). They replicate the cylinder blocks, but do not have sockets to be placed into, or the knobs to hold. They are in 4 colours- red (thick to thin, constant height), yellow (big to small), green (short and thick to tall and thin) and blue (tall to short, constant diameter) as shown in Figure 4.

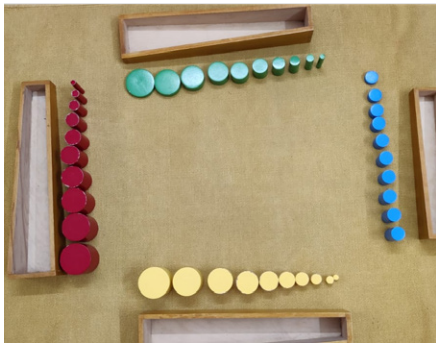

Figure 4

Figure 5

Activities similar to those described for the cylinder blocks can be performed with them. One can also match each set of cylinder blocks with the coloured cylinders.

Coloured cylinders allow for towers to be formed as they do not have knobs (An observation that children will make on their own too!). This is a great exercise for children to develop focus, patience, and an understanding of balance. Figure 5 shows two sets of coloured cylinders- the green one moving from thick and short to thin and tall, and yellow moving from big to small. They are to be placed alternately, with the same base of green-yellow pair placed one above the other.

There are exactly 4 possibilities if we vary the dimensions of any cylinder:

1. Same height, varying thickness (diameter) - Red
2. Same thickness (diameter), varying height - Blue
3. Both height and thickness (diameter) increasing - Yellow
4. Height increasing and thickness decreasing - Green

It is fascinating how Dr. Maria Montessori came up with this complete exposition of all possibilities so many years back. Such a holistic exposure prevents a learner from forming misconceptions such as bigger is thicker or smaller is shorter or thinner is shorter!

As in the previous article, we bring to you cost-effective ways of creating these Montessori materials without compromising on precision or quality. Here are the simple steps to make the knobless cylinders (Figure 6)

1. Make a net of each cylinder
2. Form the open cylinder
3. Fill the cylinder with rolled newspapers
4. Close the cylinder and seal
5. Laminate with tape

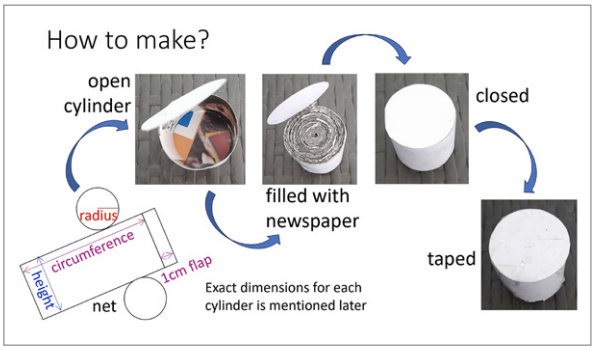

Figure 6

The exact dimensions of the nets and all other details can be accessed at [2]. In fact, there are printable nets of cylinders on that link, that you can print on an A3 sheet and you are good to go!

By incorporating Montessori materials into a child's learning environment—whether purchased or re-created—we are embracing an educational approach that prioritizes independence, hands-on discovery, and respect for each child's natural development.

Montessori education nurtures not only cognitive growth but also fosters critical life skills like problem-solving, responsibility, and confidence. The beauty of re-created materials is that they make this philosophy more accessible, allowing teachers, families and educators to create learning tools tailored to their needs without compromising on quality or intent. By integrating these materials into our home or classroom, we are taking an important step toward cultivating a generation of curious, confident, and capable individuals.

We urge you to try them out and share your experience with us. Send us photos of your creations, and let us know how it went in class with your students! You can write to us at atrightangles.editor@apu.edu.in.

Acknowledgement: This article is written in collaboration with Ms. Sudha Rao, Parijatha Montessori, Bangalore and Swati Sircar, Azim Premji University, Bangalore.

References

1. Montessori Materials Part 1 https://bit.ly/3PHeZy9
2. How to make the cylinders https://bit.ly/3PHf3hn

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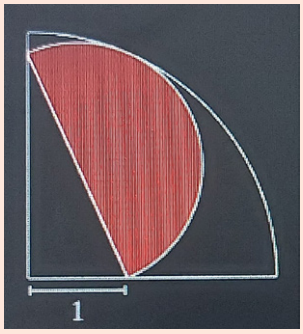
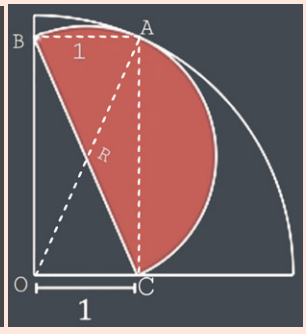


Reader Nikhil Kumar, student of class 12, PM Shri Kendriya Vidyalaya Palampur, Himachal Pradesh sent in a solution to the problem on finding the area of the red region published on page 54 of At Right Angles, November 2024 issue.



Solution:

Let O be the centre of the quarter circle (radius R) and A be the point at which the red semicircle (radius r) touches the large quarter circle. Let BC be the diameter of the red semicircle.

Since BC is the diameter of the red semi-circle, the angle subtended at A has to be 90°. Therefore, if we draw a line at C perpendicular to OC, then it has to pass through A. So $OCAB$ is a rectangle. So $AB = OC = 1$, $OB = AC$ and $OA = R$.

By Pythagoras Theorem, in ΔOAB, $OB^2 = OA^2 - AB^2 = R^2 - 1$ and in ΔABC, $(2r)^2 = AB^2 + AC^2 = 1 + OB^2 = 1 + R^2 - 1 = R^2$.

So we conclude that $2r = R$.

$$r = \frac{R}{2}$$

$$\text{Area of red region} = \pi \frac{R^2}{4} \times \frac{1}{2} = \frac{\pi R^2}{8}$$

(Note: Since the diagonals of a rectangle are equal, it is possible to directly arrive at = $OA = R = BC = 2r$)

Round and Round: Mathematics through Number Wheels

Rajkumar Kanojiya

An important part of learning elementary mathematics is mastering basic number operations such as addition, and subtraction. While repetition and practice are essential, finding creative and engaging ways to reinforce these skills can make learning more enjoyable and effective. One of the classical materials that can be used for this is *number wheels* —a classical, playful and interactive teaching-learning tool that can transform number operations into experiential activities. Number wheels are particularly useful for young learners who are building their foundational skills, but they can also be adapted to challenge more advanced students. This article will guide you through creating and using three variations of number wheels to practice and master addition. This is intended for students who have understood multi-digit addition with the help of manipulatives such as bundles and sticks.

What are number wheels?

A number wheel is a simple tool shaped like a sequence of concentric discs, divided into sections, each containing a number. The innermost disc is called the *units disc*, which contains single digits from 0-9, the second disc is called the *tens disc*, which contains the multiples of 10, from 0-90, and so on (See Figure 1). The number wheels help us keep track of place values by moving across different discs. For example, the number 27 can be marked on the number wheel as follows: Since 27 = 20 + 7 = 2 tens and 7 units, we mark 7 in the units disc (marked red in Figure 1) and 20 in the tens disc (marked blue in Figure 1). Starting from position 0, each time we complete counting all the positions clockwise on one disc, we move one position forward on the next larger disc. In simpler terms, every time we complete a full circle on the units disc, we add 10 by moving one step clockwise on the tens disc, and this pattern continues for the other discs.

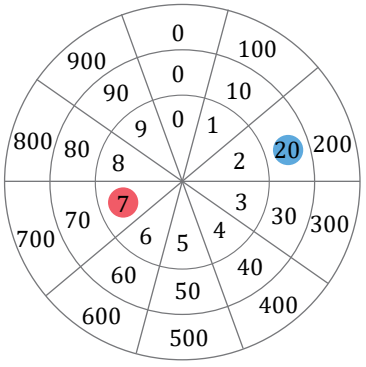


Figure 1: A number wheel with three concentric discs, where the number 27 is marked.

Using number wheels to practise elementary number operations

Let us begin with the addition of two double digits. To illustrate let us begin with finding the sum **27 + 45**. The number wheels can be drawn on the floor, where the children 'enact' the addition of two numbers. Start by choosing three volunteers: name the first child "Unit," the second child "Ten," and the third child "Hundred."

Keywords: Addition, Number wheels, Tactile learning

Step 1: We mark the number 27 on the number wheel by having the "Unit" stand at position 7 on the units disc, the "Ten" stand at position 20 on the tens disc, and the "Hundred" stand at position 0 on the hundreds disc. (See Figure 2)

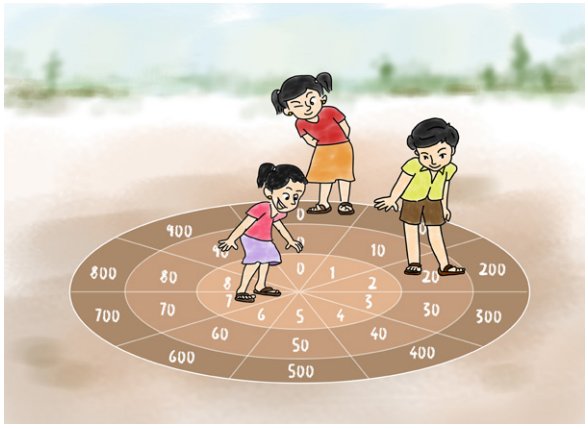


Figure 2: Marking 27 on the number wheel

Step 2: To add 45, we do the following: Since 45 = 4 tens and 5 units,

- First "Unit" moves 5 sections starting at 7 in the units disc and thus reaches position 2 in the units disc.
- Since "Unit" crossed the position 0 once, "Ten" moves from 20 to 30. This is the meaning of carrying over.
- Then we add 4 tens by moving 4 sections in the tens disc. Starting from where he is right now: from 30 to 70.

So the new positions are 2 in the units disc and 70 in the tens disc. Therefore **27 + 45 = 72** (Figure 3).

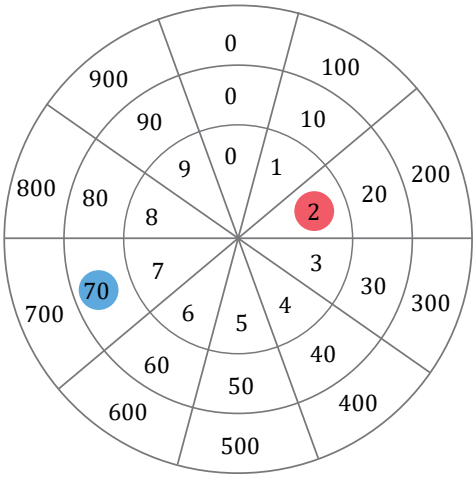


Figure 3: The final sum.

A quick set of exercises for the reader:

1. Use the number wheel to compute the following sums:
   - 7 + 5
   - 7 + 15
   - 72 + 54
2. In the example above we vaguely described the visual meaning of 'carrying over'. Explain the meaning of this process.
3. Extend the activity to explain the process of subtracting two double-digit numbers - in particular, the visual meaning of borrowing.
4. Use the number wheel to compute the following differences:
   - 12 – 7
   - 48 – 39
   - 101 – 57

Variation 2: Separated wheels

In this variation, the discs are separated and placed apart (see Figure 4). The unit disc is placed on the right, the tens disc in the middle, and the hundreds disc on the left. This setup helps emphasize the positional value of each digit, making it easier for students to connect the visual representation to numerical values.

This variation is especially useful for teaching the addition algorithm for two double-digit numbers, as it allows the process to be demonstrated clearly and step by step. Note that here the tens and hundreds discs are marked from 0 to 9, this is a sophistication that must be explained to students (i.e., for example, 5 on the units disc is 5, on the tens disc is 50 and on the hundreds disc is 500).

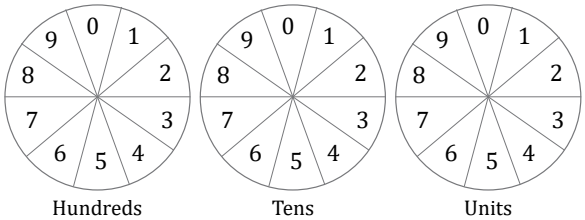


Figure 4

Variation 3: An odometer

This variation, adapted from [1], involves building a physical model of an odometer, which is used to measure the distance a vehicle has travelled in kilometres.

Materials Needed:

- A piece of cardboard (6 cm × 2 cm).
- Three paper strips (each 1 cm × 11 cm).
- Scissors, glue, and markers.

Steps to Create the Odometer: (See Figure 5.)

Step 1: Prepare the Cardboard Base

- Take the cardboard and divide it into three equal squares, each measuring 2 cm × 2 cm.
- Reading from right to left, assign each square to represent the units position, tens position, and hundreds position.

Step 2: Create the Digit Strips

- Cut three strips of paper, each measuring 1 cm × 11 cm.
- Divide each strip into 11 equal sections, each 1 cm × 1 cm.
- Label the sections with the digits 0 to 9 from top to bottom. Leave the last section blank.

Step 3: Insert the Digit Strips into the Cardboard

- In each of the three squares on the cardboard, cut a horizontal slit at the top and another at the bottom.
- Insert one digit strip through the top slit of each square and pull it out through the bottom slit.
- Roll each digit strip so that the blank section of the strip is pasted behind the '0' section. This ensures that only one digit is visible at a time through the square window in the cardboard.

Step 4: Assemble the Odometer

- Once all the digit strips are inserted, you should be able to roll the strips to display numbers from 000 to 999 in the three squares.

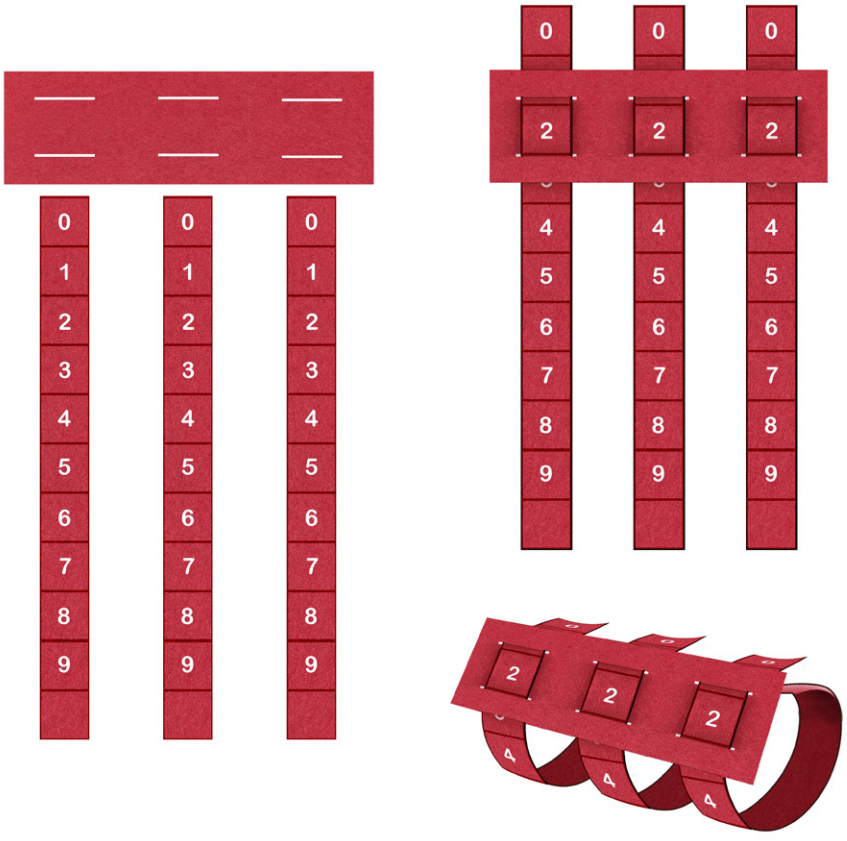


Figure 5

Using the Odometer

This model can be used for activities similar to the ones with other variations, but this one aids better with individual activity. We could also provide the children with interesting word problems involving odometers, such as:

> "When you started at home, the odometer showed 27. You travelled 45 km to reach school. What should the odometer display when you arrive at school?"

A few practical tips for using number wheels in a classroom

Here are some practical tips for using number wheels in the classroom:

- Instead of asking children to draw number wheels in their notebooks or providing them with printouts, it can be more engaging to draw a large number wheel on the floor. Children can stand on the numbers and skip or jump to the next number as they perform the operations. Other students can watch and help by pointing out the correct numbers, turning the activity into a collaborative learning experience.
- Teachers should be ready to answer questions like, "Why do we always start at 0 instead of 1?" or "What happens when we cross 0 on the tens disc?" These questions can help reinforce the concepts behind the number wheels and clarify how they work.
- Using dice can make the activity more fun. For example, you can roll a pair of dice to generate two double-digit numbers for addition or subtraction. However, keep in mind that standard dice won't generate digits from 7 to 9, which could lead to discussions about other ways to create larger numbers.
- In a creative classroom, these activities can also lead to children coming up with their own algorithms to add numbers. These TLMs can also help children enact subtraction.

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Shuffling towards Mathematics: UNO as a Classroom Tool

Sumit Kumar Pandey

UNO is a card game loved by people of all ages worldwide. Known for its vibrant colours, simple rules, and fast-paced gameplay, it has become a staple in most family gatherings and school breaks. But beyond the fun and excitement, UNO holds hidden potential as an engaging educational tool, particularly in teaching and learning mathematics. This article explores my experience of students playing an UNO multiplication game, and how teachers can creatively use UNO cards to teach mathematics.

What is UNO?

Created by Merle Robbins in 1971, UNO is a colourful and exciting shedding-type card game. The goal? Be the first to get rid of all your cards in this fast-paced game.

Here is the quick game overview:

- **Players:** 2-10.
- **Objective:** Discard all your cards to win.
- **Deck:** 108 cards in four colours (red, yellow, green, blue), with numbers 0-9 and special action cards. (See Figure 1)
- **Rules:** Match either the colour or the number of the top card in the discard pile. If you can't play, draw a card. Still stuck? Skip your turn!
- **Special Cards**
  - **Skip:** The next player skips their turn.
  - **Reverse:** Change the direction of play.
  - **Draw Two:** The next player draws two cards.
  - **Wild:** Choose the colour to continue.
  - **Wild Draw Four:** Choose the colour and make the next player draw four cards.

Keywords: UNO, card game, Mathematics games, Active Learning, Hands-on learning

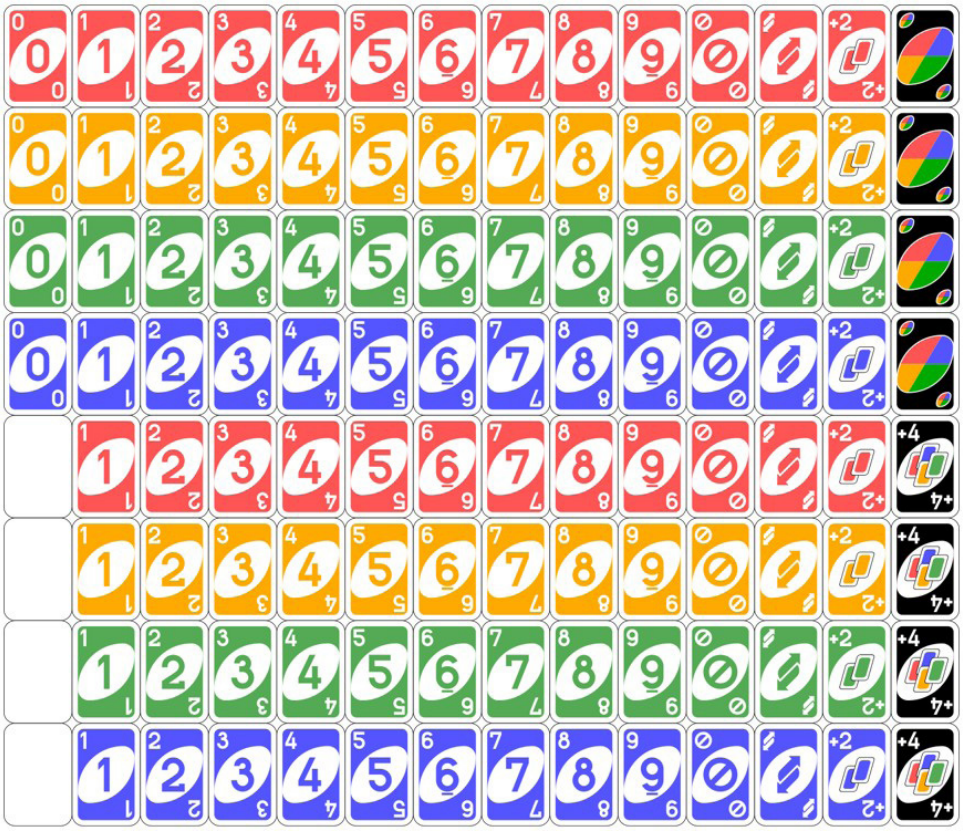

Figure 1: UNO Cards Deck (Source: Wikimedia Commons)

A UNO like multiplication game

In a resource-crunch classroom, it is of course difficult to buy UNO cards and use them. In such situations, the teacher can come up with UNO alternatives designed specifically to teach a concept in the classroom. In this section, I describe a UNO like multiplication game I designed and the classroom experience that resulted from it.

Making the cards:

- Take an ivory sheet is of dimension 56 *cm* × 71 *cm*.
- Mark the ivory sheet on every 5cm horizontally and make lines.
- Mark the ivory sheet on every 7cm vertically and make lines.
- This will give us 110 rectangles of size 5*cm* × 7*cm*, which is enough to make 2 sets of cards.
- Cut the rectangular cards and write multiplication facts on them.
- Avoid repetitions due to commutative property.
  For example, if you have made 2 × 5 then avoid making 5 × 2.
  - The multiplication fact cards will look like those in Figure 3.

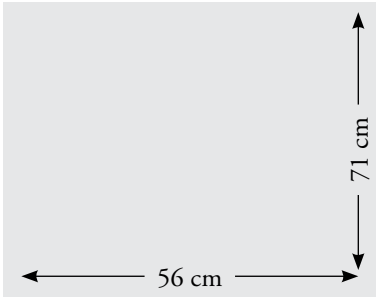

Figure 2. An ivory sheet

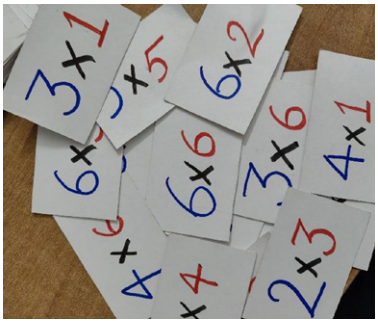

Figure 3. UNO like cards created for the game.

1 × 1	2 × 1	3 × 1	4 × 1	5 × 1	6 × 1	7 × 1	8 × 1	9 × 1	10 × 1
1 × 2	2 × 2	3 × 2	4 × 2	5 × 2	6 × 2	7 × 2	8 × 2	9 × 2	10 × 2
1 × 3	2 × 3	3 × 3	4 × 3	5 × 3	6 × 3	7 × 3	8 × 3	9 × 3	10 × 3
1 × 4	2 × 4	3 × 4	4 × 4	5 × 4	6 × 4	7 × 4	8 × 4	9 × 4	10 × 4
1 × 5	2 × 5	3 × 5	4 × 5	5 × 5	6 × 5	7 × 5	8 × 5	9 × 5	10 × 5
1 × 6	2 × 6	3 × 6	4 × 6	5 × 6	6 × 6	7 × 6	8 × 6	9 × 6	10 × 6
1 × 7	2 × 7	3 × 7	4 × 7	5 × 7	6 × 7	7 × 7	8 × 7	9 × 7	10 × 7
1 × 8	2 × 8	3 × 8	4 × 8	5 × 8	6 × 8	7 × 8	8 × 8	9 × 8	10 × 8
1 × 9	2 × 9	3 × 9	4 × 9	5 × 9	6 × 9	7 × 9	8 × 9	9 × 9	10 × 9
1 × 10	2 × 10	3 × 10	4 × 10	5 × 10	6 × 10	7 × 10	8 × 10	9 × 10	10 × 10
1 × 1	2 × 2	3 × 3	4 × 4	5 × 5	6 × 6	7 × 7	8 × 8	9 × 9	10 × 10

Figure 4: In the above table two sets can be made using a single ivory sheet of dimension 55cm × 71cm.

Rules for the game

Rules for this game are quite similar to the UNO game.

1. The cards should be shuffled properly and distributed equally to the members. It can be either 5 or 7 cards per person. The rest of the cards are placed in a Draw Pile face down.
2. The game opens with a card from the draw pile.
3. The next card should have the same product as the last card, or the next card must have one of the factors of the last card on the floor. For example, if the last card is 7×8, then the next card must either have 7 or 8 as a factor, or the same product as 7×8, and can then be discarded.
4. If both are not possible then the player draws a card from the draw pile.
5. The players should announce the factors and their product aloud while throwing the card. If the player misses this or says the wrong product and the other players catch this, the player has to pick two extra cards from the deck.
6. The player who finishes his/her cards first will be the winner of the game.

Experience of using this in class - How did the students respond?

The cards were introduced to fifth-grade students who were having difficulty with multiplication. They saw it as a fun activity and began playing. Initially, they took some time to grasp the rules, but once they got the hang of it, they realised that knowing their multiplication tables was essential to winning. This naturally motivated them. On the first day, the game took a long time to finish because they didn't know the products of the cards they had. However, as they continued playing, they gradually understood the game and started learning the tables on their own.

This activity made it easier to discuss multiplication tables with students. Instead of asking them to chant and memorise the tables, the game naturally motivated them to learn. Regular play can further enhance their computational skills. The game also helps students recall multiplication facts more effectively and improves their ability to retain information.

Dos and Don'ts for any teacher who wants to use this in class

To help young students in grades 2 and 3 understand the concept of multiplication, teachers should use real-world examples and physical objects to explain it as repeated addition. Introducing the term “times” to describe multiplication makes the concept clearer. This game can then be used to make learning multiplication tables fun and interactive. For older students in grades 4 and above, the game can serve as an enjoyable way to practise multiplication facts, encouraging them to memorise and use their tables more effectively.

Here are some dos and don’ts for teachers while students play this game:

- Ensure students understand what the cards represent.
- Brief the students about the rules of the game before they begin.
- Teachers should play the game once or twice to ensure students understand how it works.
- Motivate them during the game to find answers and participate actively.
- Arrange students in groups of 4 or 5 to play. If the class is large, create multiple groups to give more students a chance to participate.
- Ensure the game is played smoothly, keeping the students focused and engaged.

UNO as a classroom tool

In the classroom, games often serve as a bridge between abstract ideas and practical learning. They offer students a fun and interactive way to understand concepts that might otherwise feel challenging. UNO cards, with their numerical values, sequences, and logical rules, are especially suited for teaching foundational mathematical concepts. Their versatility makes them ideal for activities like flashcards or group games. Educators have designed a variety of activities to suit different age groups and skill levels, showcasing how UNO cards can effectively enhance mathematical learning. For more ideas, see the resources listed in the references [1–3].

For Foundational level (Ages 3-8)

- **Colour Sorting:** Children can sort UNO cards by colour, which strengthens their capacity to distinguish and categorise colours.
- **Number Matching:** By grouping cards with the same numbers, children develop number recognition and matching skills.
- **Number-Object-Numeral Association:** Selecting a card, counting out the matching number of objects, and identifying the corresponding numeral help students understand quantities and strengthen their counting skills.
- **Friends to 10:** This activity involves picking a card and determining the number needed to add up to ten, improving addition fluency.
- **Single-Digit Addition and Subtraction:** Drawing two cards and either adding or subtracting their values provides practice in basic arithmetic operations.

For Preparatory level (Ages 7–10)

- **Times Table Practice:** Selecting two random cards and multiplying their values helps reinforce multiplication skills.
- **Two-Digit Addition and Subtraction:** Using UNO cards to create and solve two-digit addition or subtraction problems enhances computational proficiency.
- **Greater Than or Less Than:** Comparing the values of two cards to determine which is greater or lesser fosters number sense and understanding of numerical relationships.

These activities demonstrate the adaptability of UNO cards as educational tools, making mathematics accessible and enjoyable for students at various learning stages.

Conclusion

The UNO-like multiplication game transforms the often monotonous task of learning multiplication tables into an exciting, interactive experience. Blending gameplay with educational objectives naturally motivates students to practice and master their tables while fostering quick thinking, concentration, and collaboration. The activity encourages active participation, making it far more engaging than traditional rote learning methods.

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Anno's Magic Seeds: A Review of the Story from the Perspective of Use in the Mathematics Classroom

Reviewed by Manisha Goyal

Cover Page of the bilingual edition- Anno's Magic Seeds / Jadui Beej by BGVS, published in 2005

Anno's Magic Seeds is a children's story book written in 1992 by Mitsumasa Anno (1926-2020), a mathematics teacher and famous children's book author and illustrator from Japan. He was a trained primary school teacher and a self-taught artist who illustrated more than 300 children's books. His training and keen interest in Mathematics reflects both in the narrative as well the illustrations of the story *Anno's Magic seeds*. It makes reading this story a fascinating mathematical experience and engagement (for both children and adults).

This book is part of a series of children's books by Mr. Anno on diverse mathematical themes (Anno's Counting Book, Anno's Hat Tricks, Anno's Counting House are some other titles in this series). Originally written in Japanese, *Anno's Magic Seeds* has been published in multiple languages over the years. The English and Hindi bilingual version of this book was published by Bharat Gyan Vigyan Samiti in 2005. The story of Anno's Magic Seeds deals with the idea of generation of plants from seeds to illustrate exponential growth. This book, written long before the beginning of the trend for STEM[1] books, has mathematical ideas driving the central plot, something we see in very few books even today. In Mr. Anno's words, the story was written not to merely teach mathematics, but *to delight in the arithmetic puzzles woven in both the text and the illustrations.*[2]

The story begins with a chance meeting between a young boy named Jack and an old magician who gifts him two magic seeds. The magician asks Jack to bake and eat one seed which would keep him full for a year. He also tells him to sow the other seed to get 2 more seeds the next year. Jack not only does as he is told but continues to do so for many subsequent years, until one day, he decides to manage without eating any seed and grows both the seeds together. This is the turning point in the story. Those 2 seeds turn into 4 seeds the next year. He eats 1 of them and sows the remaining 3 seeds. Next year, he gets 6 seeds, eats 1 and sows the remaining 5 seeds. And so, the story continues. In the later years, the mathematical complexity increases as he

[1] Science, Technology, Engineering and Maths based books

[2] From author's note in the story book Anno's Magic Seeds

Keywords: Children's literature, mathematics, inter-disciplinary activities, skill building

gets married, has a kid, starts storing seeds and selling them, etc. until one day, a storm appears and destroys everything, leaving Jack with just 10 seeds to start with again.

The story is interspersed with two recurring questions for the readers.

- *How many seeds will he bury?*
- *How many seeds will grow in Jack's garden the next fall?*

This kind of story structure lends itself well to planning an interactive session for classroom teaching as there are already questions built within the story. I have used this story with both the primary and middle school children. With smaller children (grade 2-3), I simply used it to experience the joy of listening to a mathematical story. I noticed that children could enjoy the story even with the basic understanding of addition, subtraction and doubling, though I needed to skip over the more complex portions of the story.

With my class 6 students, I used this story for exploring exponential growth patterns. I planned to read the story aloud while stopping in between to let the students work out the answers to the questions. When presented with the first few questions, most children were able to solve them with ease. (e.g. 2 seeds doubling to 4 in the first year, 3 seeds (after subtracting the 1 seed eaten by Jack) doubling to 6 in the 2nd year.) After the first few years, we felt the need for some way to keep track of the number of seeds as it grows over the years. For this, we created a table as shown below.

The table helped students to notice the emerging pattern and speeded up the process of calculating the number of seeds grown and sown in the subsequent years.

(To find the number of seeds grown in a year, we need to double the number of seeds sown. But seeds sown the next year are one less than the seeds grown the previous year, as Jack eats one seed every year. So, we need to double the number of seeds sown in a particular year and then subtract one to find the number of seeds sown the next year. This is the base pattern used in the story.)

Year	Number of Seeds Sown	Number of Seeds Grown	Pattern
1	2	4	$2 \times 2 = 4$
2	3	6	$4 - 1 = 3, 3 \times 2 = 6$
3	5	10	$6 - 1 = 5, 5 \times 2 = 10$
4	9	18	$10 - 1 = 9, 9 \times 2 = 18$
5	17	34	$18 - 1 = 17, 17 \times 2 = 34$
6	33	66	$34 - 1 = 33, 33 \times 2 = 66$

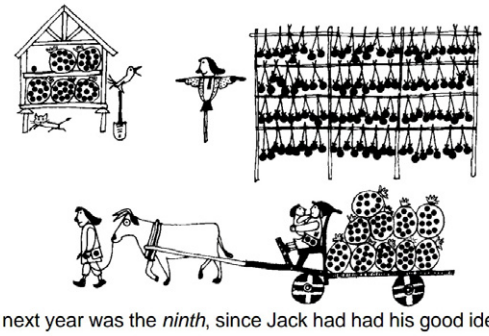
(Chart 1: Table showing the pattern in the growth of seeds over the years. Year 1 refers to the year when Jack grows two seeds for the first time.)

From the 7th year onwards, when there are slight changes to the story (i.e., Jack gets married and entertains, it requires subtracting a different number of seeds to find the seeds to be sown for the next year. At this point, some students also needed paper and pencil to calculate the answers to the problems posed in the story. But the table helped them to see that the base pattern of doubling was still the same. After the story ended, we held a brief discussion on the pattern used in the story and the strategies used by children to solve the problems. This led to the addition of the last column we see in the above table. This kind of pattern and rule recognition activity is excellent pre-algebra preparation. (See the March 2025 pullout for more ideas.)

Along with the table, the mathematically accurate illustrations with easily recognizable patterns which accompany the text of the story, also helped students when the numbers grew bigger. We see the use of multiplication arrays

and groups of tens in illustrations to make it easy to discern patterns and find the number of seeds.

For example, in this portion of the story, we can see exactly 100 seeds on the cart in neat bags of 10 seeds each, 51 seeds stacked in groups of 10 in the seed store, and 120 seeds arranged in the rows of 30 seeds each, each row further divided into 3 sections with 5 pairs of seeds each. This helps children in solving the mathematical problems that arise in the story by recognizing the patterns.



The next year was the *ninth*, since Jack had had his good idea.

In the spring a lot of sprouts came up, and in the fall a lot of seeds were made. That year their baby was born. So in the winter 3 seeds were eaten, since each of them ate one seed. Now, because they had so many seeds, they went to the market to sell 100 of them, including all those they had been keeping in the storehouse. They put 51 of the new crop of seeds into the storehouse and buried all the rest of the seeds in the field.

How many?

As a follow-up task, students were asked to work in groups and find out what would happen if the seeds could triple instead of doubling every year. Students created tables for the same in their respective groups. This helped them further explore the multiplicative patterns.

During the session, I noticed much higher class participation as compared to other days. I also observed peer group discussion on strategies for solving problems. They could also correct each other easily, in case of a mistake, without much outside help during these discussions. These are also important aspects which we need to focus on in the mathematics classroom. The use of stories like this in teaching mathematics helped me in achieving this purpose. We can also try using the same story in different ways and with different age groups to see what works the best for our students. For example, in future, while using this text, I would like to give additional challenges to the students, such as:

- What if the pattern of doubling and subtracting 1 continues for 10 years? Or 20 years?
- How many years would it take for Anno to get 1,000 or 1,00,000 seeds?

Some suggestions that were received for the foundational and preparatory stage are as follows

Foundational Stage

Children can be encouraged to connect the numbers to the pictures which show grouping in twos, fives, tens, etc. There can be worksheets on how numbers can be represented in that way. This paves the way for a deeper understanding of numbers especially with respect to divisibility. The visuals used in the story show clear links to ten-frames (both as odd-even and as groups of fives).

Preparatory Stage

The discussion on the strategies for finding the number of seeds grown and sown each year can lead children to work on pattern recognition (2, 3, 5, 9, 17…) and describing the rule (one less than double the previous number or 2n - 1) with worksheets on creating similar patterns involving 2 operations. [This can also be connected (in Secondary stage) to the teaching of Algebra.] For this, it is advisable to create a table to capture the number of seeds that are obtained, eaten, planted against the year with justification. Here, the pictures in the book can be of great help.

After this, a line graph depicting the story could also be created. Students could be encouraged to note the sharp rises and then the fall again... This also provides opportunity to may be talk about life throwing curveballs and how it can be rebuilt provided some seeds are there. What can such seeds be? Such discussions can strengthen the literature and mathematics connections.

I would also like to explore the possibility where students can design their own story books using different patterns.

This story is a great resource to be used in the mathematics classroom with children of different age and grade levels. We know that the use of children's literature helps in providing meaningful contexts for engaging with mathematical concepts. Anno's Magic seeds is a good example of such literature. Learning to observe, discuss, document, recognise and predict mathematical patterns is a key skill. The main character Jack encounters many situations-challenges, successes and setbacks) in the story which students can easily relate to and connect with their real world. Children also begin to appreciate Jack's planning and foresight for the future as well as absorb the ideas of responsibility and caring as life circumstances change. The use of this book along with other resources can definitely enrich the teaching of different concepts and the development of good life skills and positive attitudes in the classroom.

Acknowledgement: The author is deeply appreciative of the suggestions made by Ms. Swati Sircar, Azim Premji University.

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Addition Multiplication

At a visit to a school in Kollegal, we observed Class 4, where the teacher was testing the students on their understanding of multiplication facts. Most of the students answered correctly, but one student said that 2 times 3 was 5.

His classmates were quick to point out this error and the teacher added that the student seemed to have added instead of multiplying. Then the teacher asked the next student what 3 times 3 was and a whole group chorused 6 😊

Clearly the class needed some help with these errors. Send in your suggestions to AtRightAngles.editor@apu.edu.in

Manipulatives for Whole Numbers: What to use, When and Why?

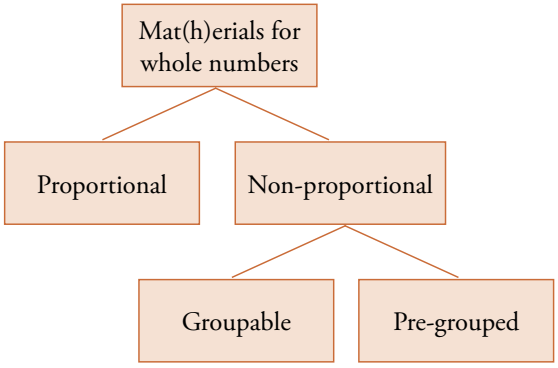
Reviewed by Math Space

Many teachers have questions about the manipulatives which are used to teach concepts associated with whole numbers such as place value, comparison, arithmetic operations, etc. This article is a comparative study of these manipulatives and addresses issues such as which manipulatives to use when and how.

At Right Angles has reviewed several manipulatives that can be used to teach place value and comparison of whole numbers as well as arithmetic operations on them. These manipulatives include (i) arrow cards, (ii) Ganitmala, (iii) counters, (iv) ten-frames, (v) flats-longs-units (FLU), (vi) Diene's blocks and (vii) static beads (in the order of appearance in various issues of this magazine. In addition, bundle-sticks are well-known and are widely used. There are a few other manipulatives which are used popularly – (a) abacus and (b) notes and coins. Several of these have entered the mathematics textbooks of Foundational (Class 1-2) and Preparatory stages (Class 3-5) of NCERT and a few states such as West Bengal and Sikkim.

Types of manipulatives for whole numbers

All of the manipulatives mentioned above, except for arrow cards, represent the quantity. The arrow card in contrast builds the numerical form using units, tens etc. The rest can be broadly classified in two groups:



- **Proportional:** where the ten is clearly 10 times bigger than the one, or the hundred is 100 times the one or 10 times the ten – ganitmala, ten-frames, bundle-stick, FLU, Diene's block, static beads fall in this category, so do counters.

Keywords: Numeracy, number skills, manipulatives, features, comparison

- **Non-proportional:** where such proportionality is assumed but not observed – abacus and notes and coins fall in this category: a ₹100 note is not 10 times a ₹10 note by area/ volume or weight, the same holds for a ₹10 or a ₹1 coin.

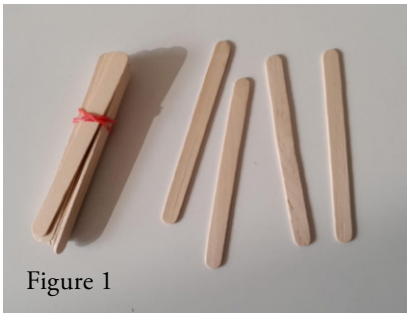


Figure 1

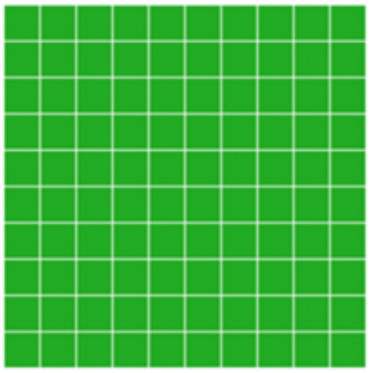
Now within proportional manipulatives there are two subgroups:

I. **Groupable:** where each unit can be part of a ten or hundred or can be on its own – bundle-sticks (and counters) fall in this category.

II. **Pre-grouped:** Where each unit, ten, hundred remains an entity by itself. It cannot be broken down further or be part of the bigger entity.

This square remains only as a unit

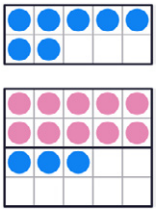
Ten/long – these squares remain only as part of a ten, never as individual units



Hundred/flat – these squares remain only as part of a hundred, never as part of a ten, or as individual units

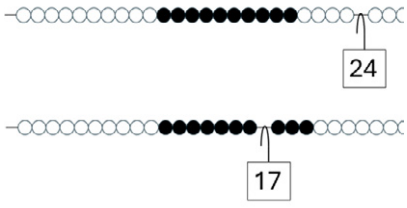
Figure 2

Interestingly, Ganitmala and ten-frames have aspects of both:



The blue counters in both 7 and 13, for example, are units whereas the pink ones in 13 are part of a full frame or ten; observe that this is independent of the position of any counter in its respective frame

Figure 3: Ten-frames



The first (leftmost) set of black beads are in a ten while representing 24, but they become units while representing 17

Figure 4: Ganitmala

	Pre-grouped aspect	Groupable aspect
Ganitmala	Beads are in groups of ten and colour-coded accordingly	Each bead can be part of a ten or be considered a single unit depending on the number we represent
Ten-frames	The 2 × 5 frame	Each counter placed can be part of a ten (if it becomes a full frame) or be considered a unit

This makes them super-useful!

Now, one must start with proportional material to build a strong foundation of understanding the base-10 structure, aka place-value. Also, since hands-on experience is very crucial for young learners, one should start with manipulatives having groupable aspects. The following table captures various aspects of these manipulatives:

Mat(h)erials	Number range	Ease of making	Uses	Virtual version	Possible extension
Counters	0-10 and more	Can be collected by the learners, these can be pebbles, buttons, seeds, etc. – good if they are identical, but not absolutely necessary	Counting, comparing, 4 operations, patterns and more	Yes	Coloured or signed ones for integers
Ten-frames	0-20, 0-50	Easy to make	Counting, automatization of single-digit addition facts, recognition of odd-even	Yes	Other frames for multiples and number patterns
Ganitmala	0-100, 0-200	Can be made by learners	Counting, **order of digits,** comparing, 4 operations and more	Not yet	4-coloured version for integers
Bundle-sticks	0-100, possibly 0-999	Can be collected and bundled by learners	Counting, **practice of grouping (in tens),** comparing, addition-subtraction	Not yet	Long coloured ones for multiplication – becomes non-proportional
FLU	0-999	Can be made by the teachers possibly with the help of older students	Representation, comparing, 4 operations, **crucial for area,** squares and square roots	Yes	Decimal FLU, algebra tiles
Diene's block	0-1000	Very difficult to make locally	Comparing, 4 operations, **crucial for volume**	Yes	Decimal version
Static beads	0-1000	Difficult to make locally	Representation Comparing, 4 operations		
Abacus	Beyond 3-digit numbers	Difficult to make locally	Representation, addition-subtraction	Not yet	Decimal version
Notes and coins	0-999	Can be made locally	Representation, addition-subtraction, word problems		

So, counters and bundle-sticks can be collected directly by the learners, while Ganitmala can be easily made by them. FLUs can also be made locally at any school but would require active supervision of a teacher and middle/high school students. Ten-frames and notes and coins can be prepared locally also. Static beads can be made locally – but it is both material intensive (about 2000 spherical beads) and labour intensive. Similarly, abacus can also be made locally with the help of a carpenter. But Diene's block maybe difficult to make unless one has access to a skilled carpenter[1].

Also, note that, learners must have access to the mat(h)erials at the initial stage of learning. So, having just one set in the class may not suffice. Students do learn by observing. But to make it deep enough, they must engage with the mat(h)erials themselves. So, ideally there should be minimum 6-8 sets of materials in a class of 30 allowing 4-5 learners per set of mat(h)erials.

Therefore, it is difficult to have adequate Diene's block and Static beads. The same maybe true for Abacus.

[1] Diene's block: The unit should be a small cube, say 1cm × 1cm × 1cm, the rod (or ten) should be 10 times a unit, i.e., say 10cm × 1cm × 1cm, the plate (or hundred) should be 10 times a rod, i.e., say 10cm × 10cm × 1cm and finally the big cube, the thousand, which should be 10 times a plate, i.e., say 10cm × 10cm × 10cm. If the unit is a bigger cube, then the rest should be bigger proportionately.

Stage-wise

Counters, bundle-sticks and Ganitmala are very important for **Foundational stage** to get started on counting, bundling in tens and playing with numbers. As a groupable material, bundle-sticks provide the much-needed concrete experience of bundling in tens, and then forming a bigger bundle whenever ten of a kind is there. [So, 10 tens make a bigger bundle, hundred.] Ganitmala shows that tens are on the left and ones on the right, and thus associating (in a 2-digit number) the left digit as ten's and the right one for units/ones. A 200-bead Ganitmala in 4 colours (2 contrasting colours showing 0-100 and 2 more such colours showing 100-200) takes it forward to show that the hundred's digit should be the leftmost. These malas are also manipulative versions of the number line with many related virtues (Figure 5). Ten-frames may not be as crucial as these three but triggers some important mental images.

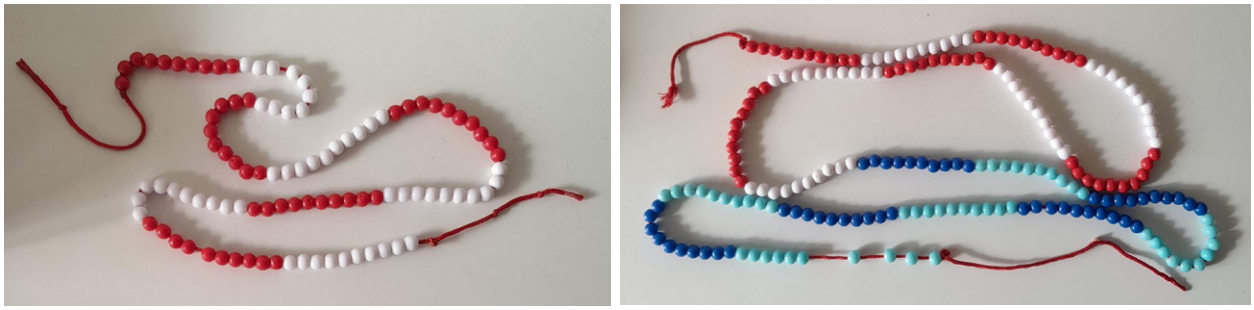


Figure 5: Ganitmala and double Ganitmala

At the **Preparatory** stage, as we move beyond 2-digit numbers, FLUs become more useful than bundle-sticks in terms of ease of use as pre-grouped material. Also, the arrays with FLUs in multiplication-division are pre-requisites for several concepts later on, including (but not limited to) perimeter and area.

As we move to 4-digit numbers, it becomes difficult to work with proportional mat(h)erials. A teacher can easily show a thousand made by joining 10 hundreds, to give a sense of how big 1000 is. But it is practically/logistically impossible to use it to compare any two 4-digit numbers or for operations etc. in a regular sized class. This is where non-proportional mat(h)erials can help. Unfortunately, ₹1000 note is no longer there. So, notes and coins can no longer help in this. Abacus can help, especially with respect to number-structure (place-value) and addition-subtraction.

One must also remember that, by the time a learner reaches 4-digit numbers, s/he should have developed adequate understanding of the base-10 structure (aka place-value) and should be able to deal with 4-digit numbers without manipulatives.

In short

Ganitmala = 1D base 10, FLU = 2D base 10, Diene's blocks = 3D base 10

- Counters are crucial since one learns to count with them, are super easy to obtain, even make.
- Bundle-sticks are also crucial since they provide the experience of bundling in tens and are easy to collect/make.
- Ganitmala is very good since it connects to number line and provides the association for the order of the digits, and is easy to make.
- FLU is excellent as the 2D base-10 blocks – 2D makes it more useful and easier to make locally in enough quantities

Therefore, the above 4 mat(h)erials are **very highly recommended** and in adequate quantity, i.e., one set for every 4 students.

- Ten-frames are easy to make and have the virtues mentioned earlier.
- Notes and coins can help by contextualizing use of numbers in real-life, especially w.r.t. buying (and selling) and can be made by students.

The above 2 matherials fall in the category of **good to have.**

- Static beads need a lot of effort and material (beads) to make. But they provide conceptual clarity.
- Diene's block demands specific craft skills and precision in terms of making.

These two matherials may be used as demo sets. Abacus – non-proportional, therefore doesn't help with conceptual clarity, might be useful for some learners who are struggling with some concepts in Class 5 or higher grades; and is not easy to make. So, **not recommended…**

Arrow cards help with unpacking the base-10 structure and are therefore, super-useful, but should be combined with some proportional mat(h)erials. They can be made locally and more easily than FLU. So, they should be used in adequate quantities.

Ganitmala, ten-frames, bundle-sticks, FLU and of course counters have entered NCERT math textbooks along with arrow cards. A few states such as Sikkim had introduced these in their textbooks previously. Abacus had entered West Bengal state textbooks. We encourage the reader to explore them – relevant links are included below.

Reference

1. Arrow Cards: https://bit.ly/42ZuwRX
2. Ganitmala: https://bit.ly/4hRl9rs
3. Counters: https://bit.ly/3EzT7m2
4. Ten-Frames: https://bit.ly/4hXAtCU
5. Flats-Long-Units (FLU): https://bit.ly/430USCK
6. Dienes Blocks and Static Beads: https://bit.ly/3Qjnb8a
7. NCERT textbooks: https://bit.ly/4jSCn9H
8. Sikkim textbooks: https://bit.ly/4aZQkPl
9. West Bengal textbooks: https://bit.ly/410ikNU

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Call for Articles!

At Right Angles is a quality resource dedicated to mathematics education in India's public education system. It is specifically designed for teachers and teacher educators at the foundational, preparatory, and middle school levels.

We invite articles from mathematics teachers, educators, practitioners, parents, and students. If you are looking for a platform to contribute articles that support and enhance the learning experience of mathematics particularly for students approximately in the age group 6-14 years , we welcome your submissions.

Suggested Topics and Themes

Submitted articles should focus on curricular content applicable to Classes 1-8 and could:

- Explain and illustrate themes and topics outlined in the National Curriculum Framework for School Education 2023 (NCF-SE 2023).
- Specifically address challenges discussed in the NCF-SE 2023.
- Be substantiated accounts of the history of mathematics or the history of mathematical thinking.
- Include innovative worksheets or methods to engage students in drill and practice.
- Describe real-life applications of mathematics relevant to the child's context.
- Describe interdisciplinary activities or projects.
- Review puzzles or games with a practical connection to the syllabus.
- Offer guidance on selecting relevant content, including online resources.
- Develop pedagogical strategies for foundational numeracy as well as computational thinking.
- Assist teachers in implementing differentiated teaching practices.
- Review of Teaching Learning Material (TLM) or describe how to use local context, and local TLM in the math class.
- Provide material to help students bridge gaps in conceptual understanding.
- Address issues in assessment.
- Suggest ways to identify and address misconceptions in mathematics learning.
- Offer a list of problems along with discussions on their solutions and problem-solving strategies that are not commonly found in textbooks.

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Articles may be sent to
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A RESOURCE FOR SCHOOL MATHEMATICS

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PATTERNS & PRE-ALGEBRA

BY PADMAPRIYA SHIRALI

A publication of Azim Premji University

PATTERNS AND PRE-ALGEBRA

Mathematics is the study of patterns - in numbers and in geometry, more commonly - but also in the most unexpected places.

Patterns are beautiful and catch our attention. All of us notice them either in our surroundings, in clothes, in constructions, and so on.

How do we describe these repetitive patterns? Some are visual patterns which share some common characteristics. Some of these are number patterns with sequences based on addition or subtraction. Some patterns are based on multiplication and division. If we know the first few numbers, we are able to predict the next number or other numbers in the sequence.

Children have also seen them and often depict them in their artwork. They have an intuitive understanding of patterns and are able to predict what comes next.

The teaching of algebra needs to build on this intuitive understanding that students have and help them to articulate their understanding in clear, concise language. The ability to generalize patterns will build the needed scaffolding for learning algebra.

Exposure to patterns in pre-algebra and articulation of the patterns using language leads to the ability to form mathematical statements. At a later point, when variables are introduced, students learn to express those same statements using variables and operations.

Here is an example related to the pattern shown in Figure 1. 'The number of lines is 4 times the number of rocket shapes plus 2.' This same statement will be expressed later as 'l equals 4 times n and 2 more.' (where l stands for the number of lines, n stands for the number of rocket shapes) or '$l = 4n + 2$'.

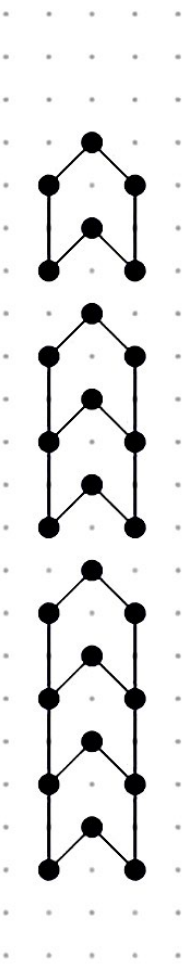


Figure 1

Keywords: Pattern, sequence, rule, connection, communication, language.

Pattern 1: Rockets, dots and lines

Figure 1 shows a number of rocket shapes made into a pattern with dots and line segments. Each line segment connects one dot to the nearest dot.

Number of Rocket Shapes	Number of lines	Number of dots
1	6	6
2	10	9
3	14	12
4	18	15

Let the students observe and note down the number of lines used in the first rocket shape. How many lines and dots have been drawn to make the second shape? Third shape? Students can record the information in a table format.

Students will notice that the pattern is increasing by 4 line segments each time. How many such lines will be needed to make 20 rocket shapes?

They will also notice the pattern in the number of dots.

Do they see any relationship between the number of dots and the number of lines?

Pose the question: How do the numbers in the second column (number of lines) relate to the numbers in the first column (number of rocket shapes)?

Number of Rocket Shapes	Number of lines	
1	6	$1 \times 4 + 2$
2	10	$2 \times 4 + 2$
3	14	$3 \times 4 + 2$
4	18	$4 \times 4 + 2$

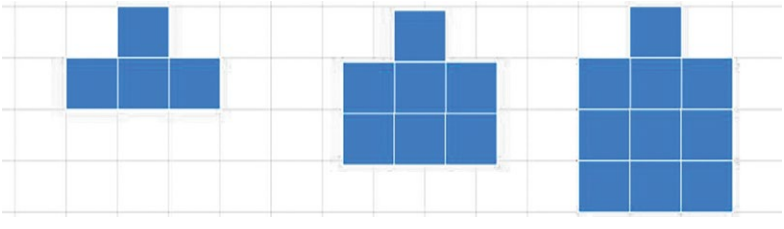
Students will see that the number of lines are not multiples of the number of shapes. However, they will be able to state that each subsequent rocket shape uses four more lines. If the students are not able to state the relationship, the teacher can ask some leading questions. Can they use the fact that the first one had two more than the rest?

Help the students to express the relationship in words.

The number of lines is 4 times the number of the rocket shape and 2 more.

Pattern 2: Increasing blocks

Let the students note down the number of squares in the table to discover the relationship between the number of each block and the number of squares it is composed of.



Block 1 Block 2 Block 3

Figure 2

Block	1	2	3
Number of squares			

If the number of squares in this sequence is 67 what will be the number of the block?

Pattern 3: Faces

Here is a series of faces made with dots and lines.

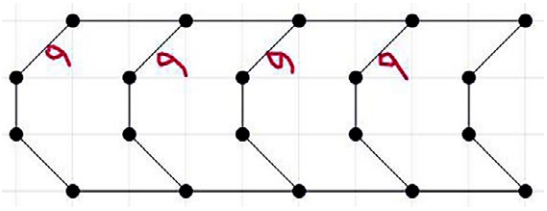


Figure 3

Students can make a table to record the information of the lines and dots used and find the relationship between the number of faces and the number of lines used. What is the connection between the number of dots and the number of lines used? How many lines will the complete figure have if there are 7 faces in it?

Pattern 4: Expanding shapes

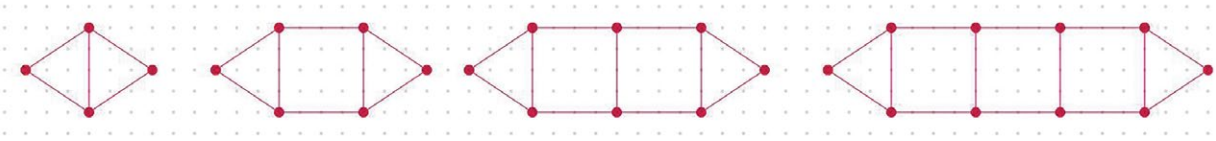


Figure 4

Create a table for these figures to discover the relationship between the number of the shape and the number of lines.

Pattern 5: Growing squares

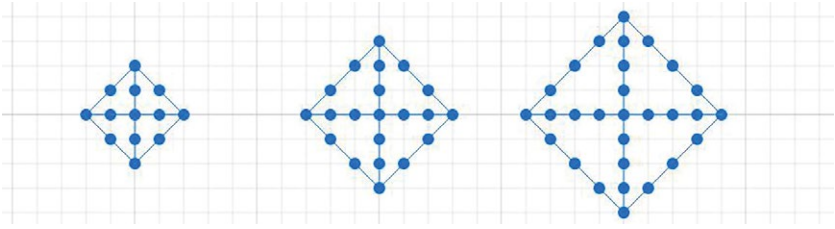


Figure 1 Figure 2 Figure 3

Figure 5

What is happening in these figures? Let the students express their understanding of these figures. Answers will vary depending on their observations.

Some students might count the dots, and some might count the lines.

Can they identify the dots that are repeating in all the three figures?

Figure Number	1	2	3
Number of dots			

How many are they? Will that number appear in the relationship of the number of dots to the number of the square?

What is happening to the other dots?

Students can count the dots and observe how the dots are increasing as the size of the square increases.

Pose the question: 'If the number of dots is 77, what will the number of the square be?'

Can the students create a figure that can be repeated and extended and verbalise the pattern?

Pattern 6: Tables and stools

Here is a classroom arrangement of tables and stools.

How does the number of stools increase if the pattern continues?

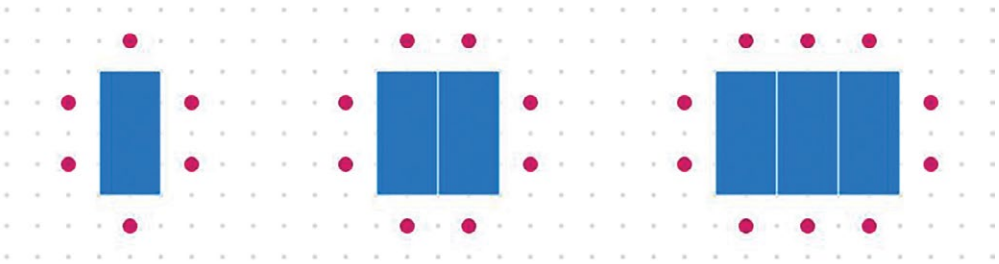


Figure 6

Number of tables	1	2	3	4
Number of stools	6			
Relationship	....			

Pattern 7: Designs with squares

How does the number of dots increase if the pattern continues?

How many dots will be there in the tenth figure in this pattern?

Figure 7

Pattern 8: Clothes and pegs

Clothesline problem: How many pegs are there in relation to the number of clothes? Will this relationship continue if the number of clothes increases?

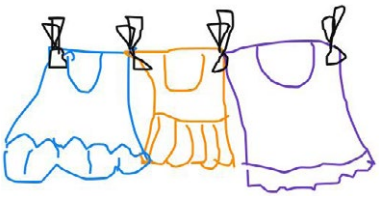


Figure 8

Pattern 9: Triangles and Lines

Look at the triangles within the orange triangles to answer the following questions.

How many white lines are used to make the inner triangles in Figure 9(1) ?

How many white lines are used to make two rows of triangles in Figure 9(2)?

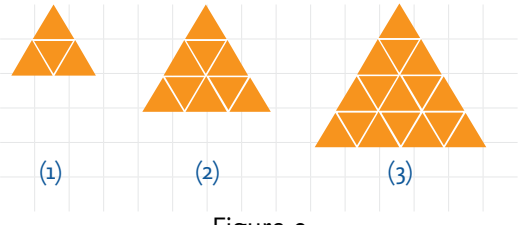


Figure 9

If we extend the pattern, how many white lines are used for three rows of triangles? How many white lines will be used to make 4 rows of triangles?

It is difficult to see a direct connection linking the row number and the lines used. However, a pattern can be found by looking at the additional lines used.

What is the relationship between the row number and the number of lines used? In the first row it is 1×3.

In the second row it is $1 \times 3 + 2 \times 3$

In the third row it is $1 \times 3 + 2 \times 3 + 3 \times 3$.

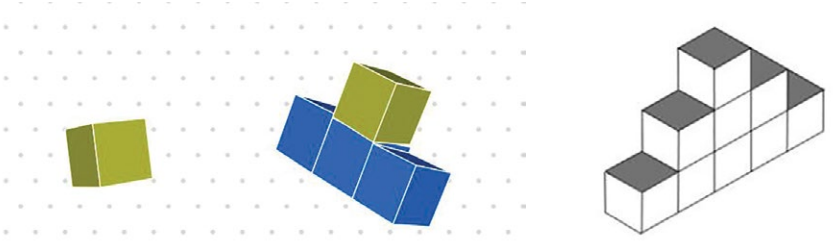
Hence, in the fourth row it will be .....

Rows	Number of lines	
1	3	3
2	9	3 + 6
3	18	3 + 6 + 9
4		

Pattern 10: Up and down staircases

In this pattern the first figure has 1 block. The second figure has 4 blocks. To continue the pattern, how many blocks are needed to make the third figure?

Can the students build a table and study the incremental increase at each stage to spot the pattern?



(1) (2) (3)

Figure 10

Pattern 11: Patterns and rules of counting numbers

Let us look at sets of three consecutive numbers:

6, 7, 8 11, 12, 13

Figure 11

What patterns can we find in these sets?

6 + 8 is 14 and 14 is double of 7.

Does this hold for the other set?

11 + 13 is 24 and 24 is double of 12.

Will this happen for all sets of three consecutive numbers? Let the students check and see. Why does it happen? How is 6 related to 7? By how much is it less? How is 8 related to 7? By how much is it more?

Can they now explain why the sum of 6 and 8 has to be double of their middle number 7?

What if we multiply the numbers 6 and 8? $6 \times 8 = 48$. If we multiply 7 by itself, it is 49. 6×8 is one less than 7×7.

Does this work for 11, 12, 13?

$11 \times 13 = 143$. $12 \times 12 = 144$

11×13 is one less than 12×12.

Ask students to make a dot array for 6×8 and 7×7 to understand the connection.

In the array of dots showing 6×8, if we remove one column of 6 dots, and turn it into one more row , we will have 7 columns of 7 dots each except for the last column.

Ask the students to show why 11×13 is 1 less than 12×12 using dot arrays.

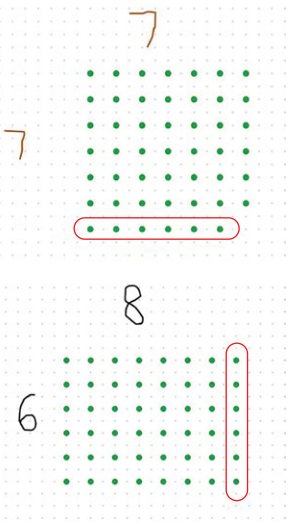


Figure 12

Pattern 12: 4 consecutive numbers

Students can now study sets of 4 consecutive numbers.

9, 10, 11, 12 4, 5, 6, 7

Figure 13

How do the numbers 9, 10, 11, 12 relate to each other? Is there a similar relationship between 4,5,6,7?

Students can try summing different pairs in these sets to look for relationships.

They may try adding 4 to 5 and 6 to 7.

Sum of 4, 5 is 9 and sum of 6, 7 is 13. 9 is 4 less than 13.

What if they try the same operation with another set of 4 consecutive numbers, say 9, 10, 11, 12.

Sum of 9, 10 is 19 and 11, 12 is 23. Again 19 is 4 less than 23.

Can the students give a justification for this result?

What if they add 4, 7 and 5,6. Sum of 4,7 is 11 and 5,6 is also 11. Will the same happen for 9,10,11 and 12?

How can they express their findings using statements?

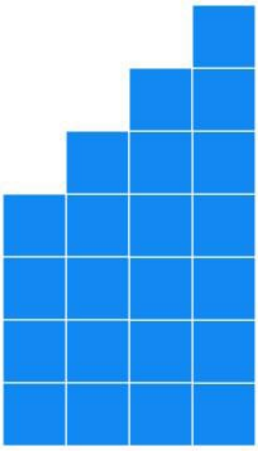


Figure 14

Pattern 13: Sum of 4 consecutive numbers

Let them add the first 4 consecutive numbers 1 to 4. They get a sum of 10. What will they get if they add the numbers from 2 to 5? 3 to 6? 4 to 7?

What pattern do they notice in the sums? Can they explain the reason for the pattern? Can they express the pattern as a statement?

Let the students add the numbers and record the sum.

Sum	1 to 4	5 to 8	9 to 12	13 to 16
	10			

What pattern do they notice here? Can they explain why it works?

Pattern 14: Consecutive multiples

Here is a set of 4 consecutive multiples of a number that have been increased by 1:

13, 16, 19, 22

This sequence is a multiple of __ , increased by 1.

Find the number which has been multiplied and the number which has been subtracted to give the following numbers.

22, 26, 30, 34

The relationship can be expressed as a sentence.

This sequence is a multiple of __ , decreased by __.

It can also be considered as a sequence that is a multiple of ....., increased by 2. Can the students explain why this happens?

Teachers can apply a rule to a series of multiples of a number and generate sequences that students can decipher. They can express the rule in the form of a statement.

How did the students work out the table and increase/decrease each time? Did they use any method?

Will the method always work?

Pattern 15: Bags of coins

Here are 3 bags with 5-rupee coins, 2-rupee coins and 1-rupee coins. Each student is allowed to pick exactly five coins from any of the bags. What coins could each one of them select to make a sum of 9 rupees? Students can express the statement as four 1- rupee coins and one 5-rupee coin makes a sum of 9 rupees.

What combinations will give a sum of 18 rupees?

Figure 15

Pattern 16: Square counting

How many squares are in each figure?

How will you describe the rule for the number of squares in this series for example?

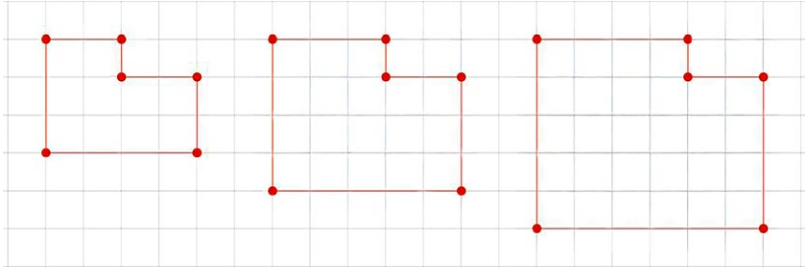


Figure 16

Pattern 17: Square designs

What fraction of the squares are shaded in blue in each figure? What fraction of the squares are shaded in grey in each figure? Are the fractions increasing or decreasing?

How does the number of blue squares increase in this series? How does the number of grey squares increase in this series?

How many blue squares will be in the 10th figure? How will you describe the sequence of the number of blue squares?

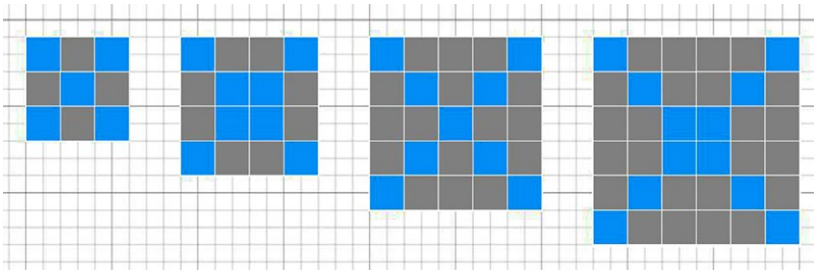


Figure 17

How are the blocks in Figure 18 increasing?

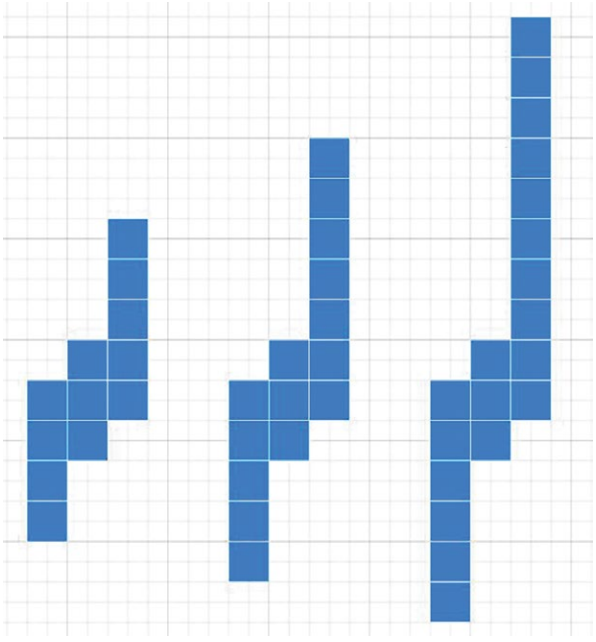


Figure 18

Challenge question: Flower pattern

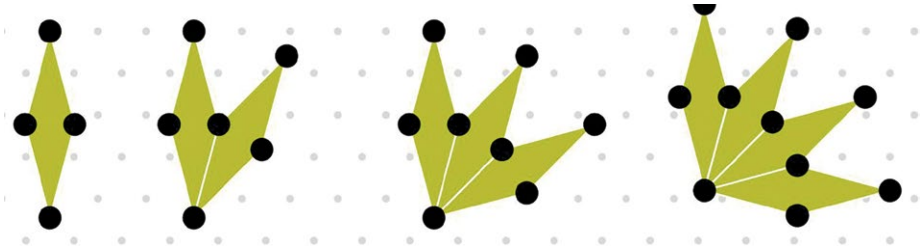


Figure 19

Here is a flower pattern. How are the figures increasing?

How many dots to create a closed figure? How many petals will there be in the shape altogether?

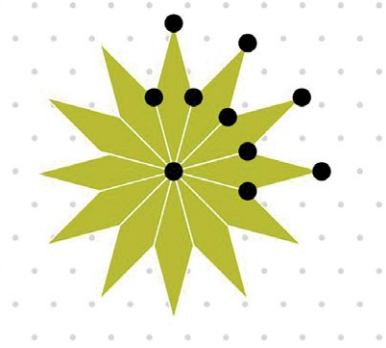


Figure 20

Figure 20 gives the answer

Pattern 18: Number line movements

On this number line an engine goes forward by 3 steps and moves backwards by 1 step in one round, then moves forward by 5 steps and goes backward by 2 steps in the second round, then moves forwards by 7 steps and goes backwards by 3 steps in the third round. The pattern goes on repeatedly forwards and backwards. Where will the engine be after 8 rounds? How will you describe this sequence?

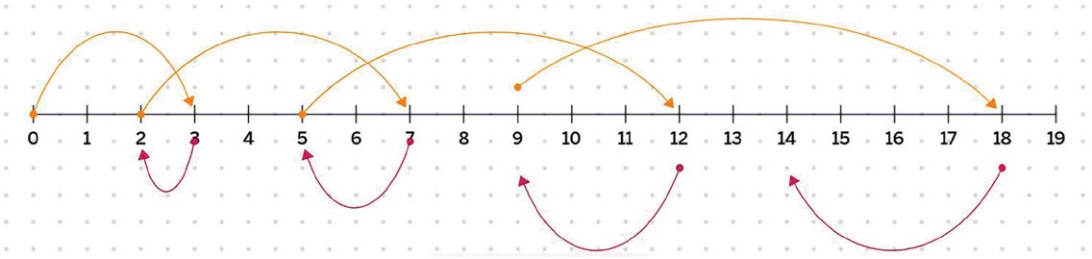


Figure 21

Pattern 19: Fractions in squares

In these series of squares the top row of the first square is shaded fully. The second row has one less shaded square than the top row, and so on. What fraction of the squares are shaded? What pattern do you notice in the fractions? How are the denominators increasing? How are the numerators increasing? How will you describe this sequence?

Is the fraction of shaded squares increasing or decreasing?

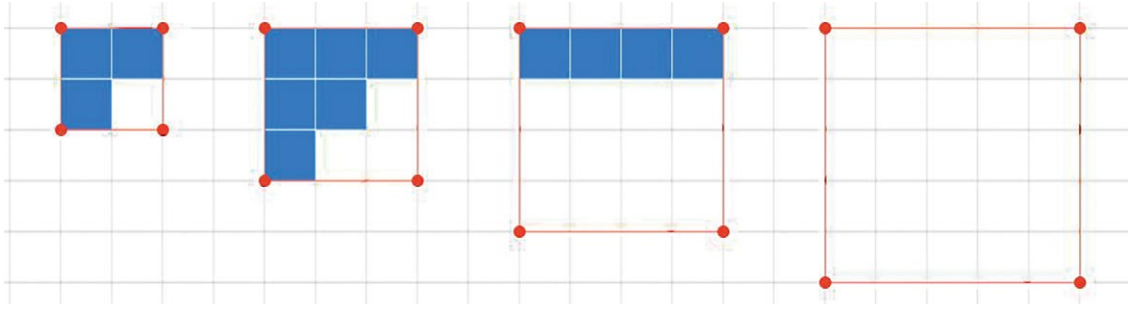


Figure 22

Pattern 20: Tables

What is happening in these tables? Try with a combination of operations to figure out the rule. How do you describe the rule?

Input	Output
7	41
9	51
3	21
6	36

Input	Output
8	63
13	168
11	120
15	224

Pattern 21: Grids

Here is a grid made up of a pattern. The numbers go horizontally from left to right in steps of 6. The numbers go vertically down in steps of 5.

What is the rule for the numbers that are shaded with the same colour in this grid?

0	6	12	18	24
5	11	17	23	29
10	16	22		
15	21			

What is the rule for the numbers that are shaded with the same colour in this grid?

0	6	12	18	24
5	11	17	23	29
10	16	22		
15	21			

What is the rule for the numbers that are shaded with the same colour in this grid?

0	6	12	18	24	29
5	11	17	23		
10	16	22			
15	21				

Fill the grid with numbers and colour the numbers according to a rule. Can your friend figure out the rule?

0	1	2	3	4	5	6	7	8	9
10	11	12	13						

Pattern 22: Intersecting squares

How is the perimeter of the whole figure increasing?

How is the area of the whole figure increasing?

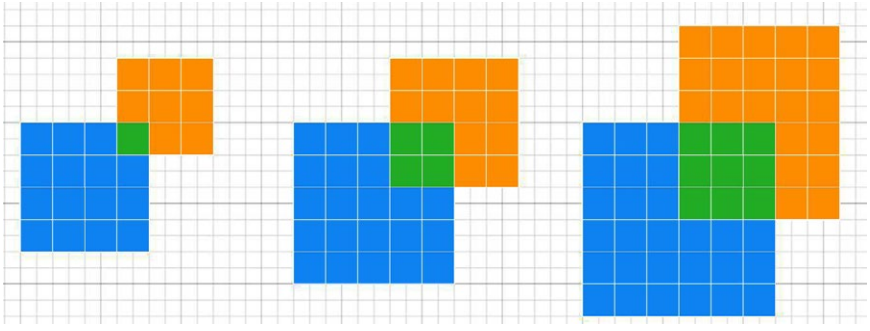


Figure 23

Pattern 23: Graphs and dots

What rule will connect dots of the same colour?

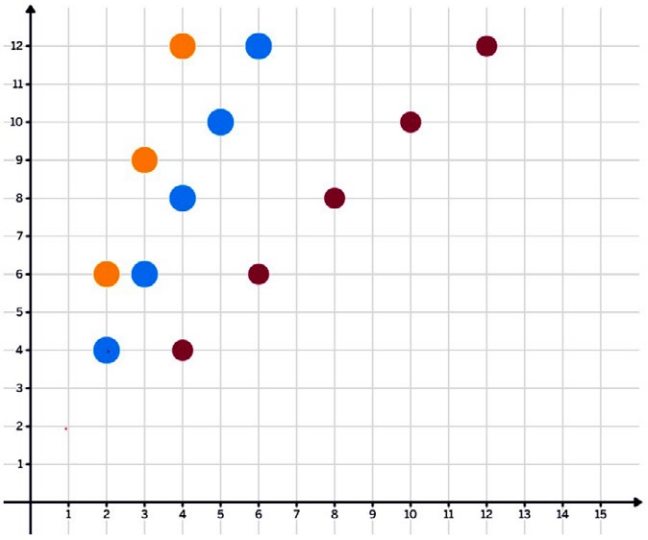


Figure 24

Pattern 24: Magic squares

Here is a magic square. What is the magic sum of this square?

What patterns do you see in the number arrangement in the square?

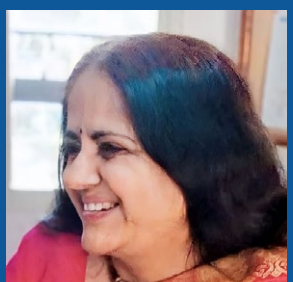
13	**8**	**15**
14	**12**	**10**
9	**16**	**11**

Figure 25

How do you describe the numbers at the opposite corners in relation to the number at the centre?

How do you describe the numbers: (a) above and below (b) to the left and right

in relation to the number at the centre of the square?



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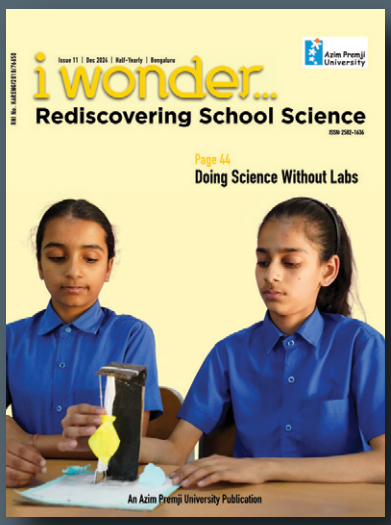

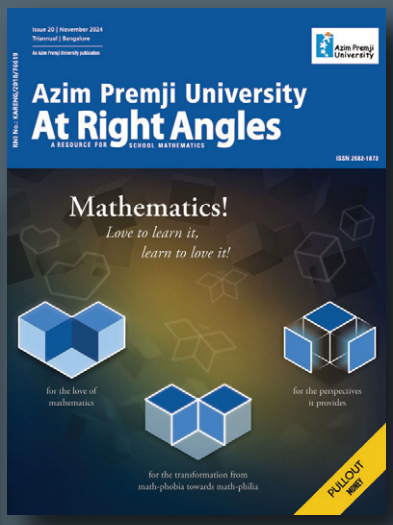

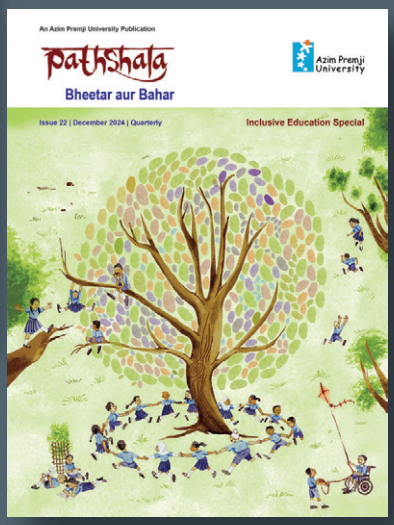

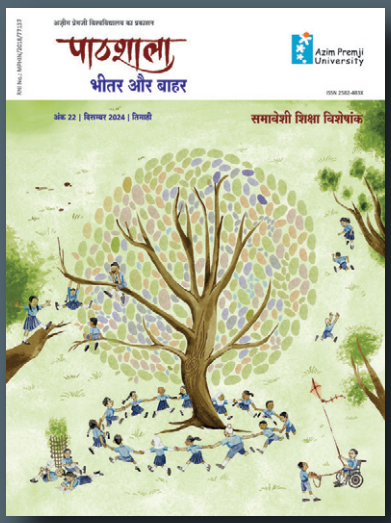

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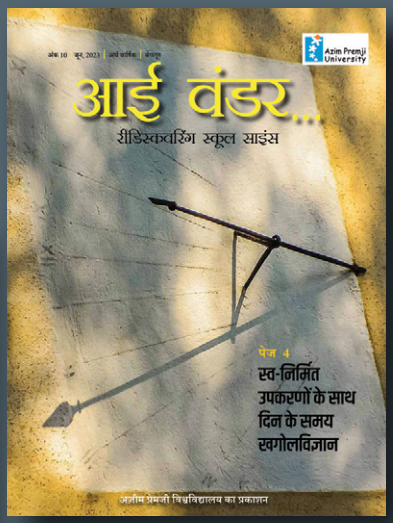

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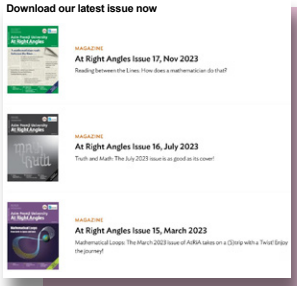

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