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At Right Angles

A RESOURCE FOR SCHOOL MATHEMATICS

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Have you ever felt that finishing your examinations is an experience similar to successfully walking over hot coals or escaping from a fiery furnace? Have you submitted your students to a similar trial by fire? Have you wished that things could change? Can an end of year summative assessment be crafted to be an authentic assessment of a child's attainment of prescribed competencies? Why not? We don't have to assess a child's endurance or performance under stress, and definitely not at the primary stage. Assessment is necessary but not a necessary evil- how can we make it a natural process which brings out the best that a child can do? How can we facilitate and support students as they negotiate the challenges they face- the mountains they need to climb?

From the Editor's Desk . . .

Dear Readers,

Spring is in the air! It's March and the cold of winter is giving way to fresher breezes, blooming buds and the prospect of the academic year ending and the summer holidays beginning. But wait- before those endless days of leisure, there is one final hurdle, that dreaded final examination, the summative assessment that is the bottom-line for the year of learning. With teachers quickly picking up the use of and need for new techniques for formative assessment, the usual pencil-and-paper mathematics tests have evolved into more fun-but-as-rigorous tasks. These serve to inform the teacher, the student and the parents about what the child has learnt, and the areas in which more reflection and effort is required on all their parts. However, that final examination seems to be a different cup of tea. Does it have to be so? In this issue, Aanchal Chomal, Shilpi Bannerjee, Anusha T and Reshma Krishnan examine the need for *Summative Assessment at the Preparatory Level* and ways to administer it in a way that is authentic and age-appropriate. Perhaps the planning for the next academic year can include ideas from these articles.

Kanchana Suryakumar shares her experience with the *Tricky Truth about Visualising Fractions*, and the article is backed by a worksheet designed by Kshama Chakravarthy. This should help you to design more questions which strengthen the importance of defining the whole, when teaching fractions. What's next? Our alumni Garima Bhatt *(Classroom: Time)* and Asma Memon *(Joy of Mathematics: Net of a Cube)* have delighted us with their contributions. We also have Swati Sircar's worksheet on *Cuboids and their Nets* in response to a difficulty encountered by a student teacher during her practice teaching. The Joy of Mathematics section has a lovely poster which is sure to spark thought – put it up on the bulletin board and wait for the students to exercise their observation skills, make conjectures, communicate their thoughts and argue their points- in short, behave like little mathematicians. Teacher notes are given in the accompanying page.

A *Review of the nRich website* will point you in the direction of rich resources and Padmapriya Shirali pitches the Pullout at the Foundational and Preparatory stage with tips on *Introducing Counting*.

The March 2026 issue is a special one for us- for the first time, the articles appeared first on the digital edition, followed by the print issue. Moving with the times, but keeping our eyes firmly on good mathematics pedagogy built right from the primary level. Let us know what you think.

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At Right Angles is a publication of Azim Premji University which provides quality mathematics learning resources for school teachers. It intends to facilitate more experiential and meaningful teaching-learning processes, not just inside the classrooms but also in the broader context of school processes. To celebrate purposeful and passionate teaching, At Right Angles showcases practical insights grounded in the realities of India and its diverse communities.

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Making Summative Assessments Meaningful at the Preparatory Stage

Aanchal Chomal & Shilpi Banerjee

While ***formative assessments*** take place regularly and are integrated into everyday classroom interactions, ***summative assessments*** are carried out at the end of a unit, or during mid-term or end-term periods. These summative assessments should provide a clear picture of the child's readiness to progress to the next grade or the subsequent unit in the syllabus. It is important to note that assessments are objective and guided by the prescribed competencies (stage specific learning achievements that are observable and can be assessed systematically [2]). If any learning gaps are identified, it is essential that schools offer targeted support and bridging opportunities to help the child. But very often, summative assessments are the bottom line, the labelling factor with which the child goes into the next class. With such high stakes at risk, the final examination becomes an annual torture session for children as well as parents. Can something be done differently? We begin this article with a look into a classroom.

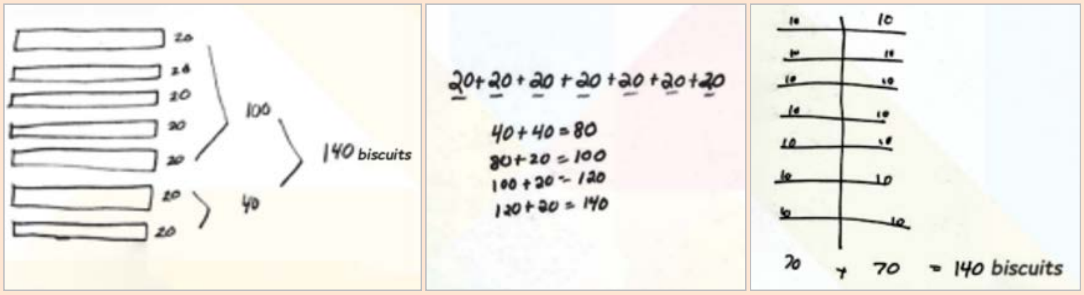
The following vignette illustrates the observations of a teacher administering formative assessment in her class and the contrast she notices during summative assessment.

A teacher's reflections on learning and assessment

Mrs. T teaches Mathematics in Class 3. She wants her students to be able to visualize arithmetic operations and relationships among them using math vocabulary. She uses the opportunity of an upcoming picnic to pack biscuits for it and makes 7 packets, each containing 20 biscuits. Next, she asks them to show how they would find the total in different ways:

Abhiraj starts with a sketch of 7 biscuit packets with 20 biscuits each. He then groups the packets and finds the total in three steps. Parth uses numerals which he then adds cumulatively, again by grouping. Mala splits each pack into 2 and then adds. Mrs. T walks around asking, "How many twenties are there?", "How many biscuits in each packet?", "How did your method help you arrive at the total quickly?" She encourages students to talk with assigned partners and keeps a ear open for correct usage of mathematical vocabulary. She gives immediate feedback, pointing out when a representation is correct, asking guiding questions when it is unclear and pointing a student to a classmate's correct representation when it is wrong. Children share their ideas, compare methods, and change or correct their approach [1]. By the end, she is aware that most students can present visual representations of numerical facts and relationships, can do repeated addition, and explain their thinking confidently using appropriate mathematical vocabulary. She is also able to identify which students can do more than this and which students need help in specific areas.

Keywords: summative assessments, formative assessments, competencies, preparatory stage, feedback, child-centred, low stakes, NCF-SE 2023, criterion referenced.



Abhiraj's representation   Parth's representation   Mala's representation

Figure 1

Later that month, Mrs. T conducts a summative assessment on arithmetic operations for the same class using a worksheet. As the students received the paper, a few of them looked nervous. The feeling of being 'tested' made them anxious, in contrast to the playful, hands on learning they had experienced earlier. Mrs. T notices their tension and gently reminds them to take their time and think through each problem carefully.

Seeing the nervousness of her otherwise confident students, Mrs. T begins to question whether summative assessments should exist at all at the Preparatory Stage. Formal summative assessments play an important role, but high stakes summative assessments might induce stress and shift the focus from joy of learning to performing for grades. Children are still developing their cognitive skills such as comprehension, problem solving, reasoning, etc., and affective skills such as cooperation, teamwork, self-awareness, etc. So far, she has conducted summative assessments mainly through traditional written examinations. In contrast, her experience with informal, formative assessments in the classroom has been very positive. She can identify misconceptions, provide timely feedback and make early adjustments in her pedagogy to support learning. Could there be a middle ground where summative assessments are low stakes and seamlessly integrated with ongoing formative assessments?

The ***Preparatory Stage*** (Grades 3–5, ages 8–11) is a crucial phase in a child's learning journey, bridging the playful experiences of the Foundational Stage and the more formal learning of the Middle Stage. Children begin engaging more deeply with subjects such as Languages, Mathematics, Art, Physical Education, and The World Around Us, transitioning from "learning through play" to structured experiences. At this age, they are naturally curious, increasingly independent, and eager to explore ideas logically. Hands-on activities, collaborative tasks, and guided reflection support their developing capacity for abstract thinking and understanding of structured concepts.

There is also a transition from low-stakes assessment (minimal consequences, primarily used to support learning, provide feedback, identify gaps, and guide instruction) to high-stakes assessments (carrying significant consequences for students, teachers, or schools; results may determine promotion, certification, admissions, or accountability decisions).

In this article, we explore these questions about summative assessments at the Preparatory Stage and provide suggestions for making them more effective and meaningful. Using examples, we unpack complex terms such as cognitive and affective skills. Different methods of administering summative assessments are also suggested.

Nature of summative assessment

Research and policy evidence [3] suggest that summative assessment can play an important role at the Preparatory Stage if it is carefully designed and used alongside ongoing formative practices. Formative assessment supports ongoing feedback, motivation, and self-regulation, while summative assessment provides reliable evidence of learning across all subjects and helps with accountability and curriculum planning. Moreover, when summative assessments go beyond rote memorization to include tasks that assess application, analysis, and higher-order thinking, they encourage deeper understanding and long-term learning.

According to the National Curriculum Framework for School Education 2023 [2], written tests may be introduced at this stage, but assessment should not rely on tests alone. Other methods, such as portfolios, peer and self-assessment, and teacher observations, should also be used to get a complete picture of student learning. When summative assessments are visualized as a broader process, they can serve several useful purposes.

Summative assessments are used to-

- Track student progress against prescribed competencies.
- Provide opportunities for students to synthesise their learning- i.e., to bring together, connect, and integrate what they have learned across a unit or topic, rather than to simply recall isolated facts. This is a higher order thinking skill.
- Determine whether students are ready to progress to the next stage.
- Communicate and report students' progress to parents, administrators, and other stakeholders.

Overall, both research and policy support a balanced approach: summative assessments add value at the Preparatory Stage when they are integrated into a varied, developmentally appropriate assessment system that prioritizes ongoing, low-stakes, formative and child-centred practices to support learning and inform teaching.

Key principles of summative assessments

1. Whether the aim is to guide teaching through formative assessment or to check progress through summative assessment, teachers need strong knowledge of the mathematics curriculum, an understanding of how students think, awareness of different assessment methods, and the ability to interpret information from many sources. This knowledge (known as PCK or Pedagogical Content Knowledge) is essential for collecting useful information and drawing accurate conclusions about student learning.

   For instance, while discussing fractions, a teacher notices that some students believe that 1/8 is bigger than 1/4 because "8 is bigger than 4." Because the teacher understands common misconceptions in math, she can quickly address this during class (formative assessment). Later, when she conducts a short quiz at the end of the unit (summative assessment), she uses her knowledge of assessment methods to design questions that check conceptual understanding, not just procedural steps. For example, instead of simply asking students to add 73 and 52, she could ask students to add any 2 two-digit numbers to get a three-digit number.

2. Summative assessment could be comprehensive and multidimensional, using a variety of methods- such as teacher observations, written work, oral responses, and performance-based tasks- to capture the full range of student learning. It could also be continuous and cumulative, i.e., incorporating short, age-appropriate written tasks alongside ongoing oral and performance-based evaluations.

   For example, at the end of a unit on basic geometry, a teacher could use multiple methods to assess student learning. She could observe students as they identify and classify shapes during hands-on classroom activities, review their written work where they draw and label shapes, ask them to explain properties of shapes orally, and also give a small project in which students create a simple 2D shape using paper or clay. Short quizzes could also be included throughout the unit to track progress. By combining observations, written tasks, oral responses, and performance-based activities, the teacher gains a comprehensive picture of each student's attainment of geometry competencies.

3. Summative assessments should assess how well students have attained the stage-specific competencies. They should not be used to label or compare children. (Such assessments are described as criterion referenced which aims to report student learning with respect to the stage specific competencies [2].)

   For example, at the end of a Preparatory Stage mathematics unit on addition and subtraction, the teacher gives a worksheet with problems aligned to the competencies. Each student's response is evaluated against the desired competencies, such as solving two-digit subtraction or word problems. The results show what each child has mastered and what needs more practice, **without ranking students against one another.**

4. Summative assessments should be conducted internally by teachers so that they can be appropriately contextualized according to students' backgrounds and prior knowledge. At this stage, assessment should not be an external imposition by bodies such as educational boards or school systems; instead, it should be an integral part of the learning process.

Some factors that could enable quality summative assessments

The transition to this sort of imaginative summative assessment is not easy. But it is desirable. What is needed is a systematic plan and approach towards making summative assessments truly meaningful as a process in the child's learning trajectory. It would mean doing a few things and doing them differently.

1. Teachers would need orientation on the possibilities of various tools and methods that can be used for summative assessments. Unfortunately, summative assessments have only been equated with paper and pencil tests. That mindset needs to change. While using multiple methods, the teacher has to also develop some kind of rubrics or detailed scoring guides/marking schemes to ensure objectivity in the assessment process as well as synthesize the learning of her students.

2. Synthesis of such learning can then be used for lesson planning and for working with specific children in areas where they are yet to develop mastery. We call this as formative use of summative assessments. This principle is also suggested in the NCFSE. Such a process helps in blending the summative assessments with the teaching learning process and provides direction to the formative assessments that a teacher can undertake subsequently.

   (Editor's Note: While such lesson plans are beyond the scope of this article, we plan to include such plans in follow-up articles.)

3. Reporting of summative assessments should also be done beyond marks and grades. There should be qualitative descriptions of the child's competencies that is easy to understand. For instance, with reference to Figure 1, at the end of the first 3 months in grade 3, *Abhiraj, Parth, and Mala can confidently represent 20 × 7 using pictures. Abhiraj grouped the 20s into larger place-value units (such as hundreds), Parth showed the idea through repeated addition, and Mala broke 20 into 10 + 10 to build her solution. Their strategies revealed clear understanding; however, they were still developing the mathematical vocabulary needed to explain their representations clearly.*

   Such descriptions not only indicate what a child can do, it also gives direction to future course of action.

4. Professional learning communities of math teachers may be initiated at the school level, or a region level to brainstorm ideas, build collective resources like activity and question banks. There could be periodic documentation of best practices that can be published and shared with the larger math community of educators.

5. The school should play a role in sensitizing parents to understand the capabilities of their children and that these cannot be defined by marks, numbers and grades. Very often parents demand comparison and ask how their child is doing in relation to others. This sort of mindset and expectation often limits the purpose of summative assessments to the numerical score obtained. It is imperative that schools also conduct camps and orientations for parents to understand the need for making a transition in the summative assessment processes for better student learning.

In conclusion, summative assessments at the Preparatory Stage need not be abandoned, but thoughtfully reimagined to align with children's developmental needs and the goals of meaningful learning. When designed as low-stakes, criterion-referenced, and teacher-led processes, they can complement formative assessment and support deeper understanding rather than rote performance. Integrating multiple methods of assessment allows teachers to capture a holistic picture of children's competencies while preserving the joy of learning. Such an approach also enables assessment to inform instruction, provide targeted support, and guide future learning. Ultimately, balanced and child-centred summative assessments can strengthen teaching–learning processes and ensure that assessment remains a tool for growth rather than judgment. More details on summative assessment strategies and tools can be found in the companion article titled 'A lens to look at summative assessment in Math', where most of the enabling factors have been illustrated.

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A Lens to view Summative Assessment in Preparatory Stage Mathematics

Anusha T and Reshma Krishnan

This article follows from the companion article 'Making Summative Assessments Meaningful at the Preparatory Stage' and throws light on how preparatory-stage teachers can design meaningful summative assessments that are age-appropriate, low stake, and aligned with the developmental needs of the children. It presents three sample strategies, with some suggested tools for assessment. Strategy 1 is an exploration task on measurement which can be assessed using a rating scale. Strategy 2 is a project which integrates spatial reasoning with art and is assessed using rubrics. Strategy 3 is a form of demonstration task to assess geometric reasoning with a self-assessment checklist. These strategies can complement written tests to make summative assessments more meaningful and comprehensive. This approach also provides a joyful and stress-free learning environment for the children.

The companion article presented the opinion that formative and summative assessments should be used meaningfully to help students attain grade-specific learning outcomes and stage specific competencies. Tasks should not create unnecessary stress or anxiety among students, should be feasible to implement in a classroom setting, and provide students and the teacher with opportunities to synthesize their learning, and at the same time give solid evidence of students' preparedness (gauged against the prescribed competencies) to move to the next stage of learning.

Strategies used for summative assessments that are both age-appropriate and aligned with research and policy evidence should include more than written assessments which are not sufficient to assess the competencies of the preparatory stage. Also, the reporting of summative assessments should go beyond marks and grades. There should be qualitative descriptions of the child's competencies that are easy to understand.

This article shares some sample strategies which may be used along with written assessments to make a summative assessment of the competencies attained by the student. Some scoring tools to assess them are also given. The tools given here are not exhaustive. Teachers can try out different combinations of tools for the same task [1].

As teachers engage with the activity rich Mathematics textbooks at the preparatory stage, a recurring concern is how these activities can be adapted into summative assessments that are both meaningful and feasible. The following strategies illustrate how students' attainment of competencies can be assessed as well as reported while placing emphasis on the ***formative use of summative assessments***. Each strategy is illustrated with sample activities and it is hoped that teachers can design more of them to assess the different competencies prescribed for the preparatory stage.

Keywords: summative assessments, competencies, exploration, self-assessment, projects, rating scale, rubrics, checklist.

Strategy 1: Exploration

The task described here helps to assess the competency *"Deduces that shapes having equal areas can have different perimeters, and shapes having equal perimeters can have different areas"* as described in the National Curriculum Framework for School Education 2023 [4, P.274].

The following is an exploration activity that has been designed as pair-work to be conducted at the end of the unit on measurements (area and perimeter of squares and rectangles) in Class 5. This task can be conducted in 60 - 80 minutes.

Traditionally, summative assessments are for individual work, but exploration tasks work better in small groups. Of course, if a child chooses to work alone, the teacher may decide to give him or her this opportunity. The observation rating scale (Table 2) given at the end of this strategy helps the teacher to assess each individual child.

Instructions

You have 24 squares, each of side one unit. Your task is to use all 24 squares and make as many different rectangles as you can.

Alone or in pairs, draw the rectangles formed using 24 squares, on the given grid sheet and write the measure of the length and breadth of each rectangle.

Now, each of you will make the table (Table 1) below. Without any discussion with each other, take turns to note down the length and breadth of all the rectangles made.

Figure 1: This image is generated using an AI tool

Rectangle Number	Length	Breadth	Area	Perimeter
1				
2				
3				

(Further rows may be added)

Table 1

Without any discussion, answer the following questions with the help of the table and the figures made.

1. Which rectangle has the smallest perimeter?
2. Which rectangle has the largest perimeter?
3. What is the area of each rectangle formed?
4. What changes do you notice in the perimeter when the shape becomes longer and thinner?

Note to the Teacher

- Though the task begins with pair work, students are assessed individually.
- It is not mandatory that the children use the given squares. Children who want to work without using concrete materials may be allowed to do so.
- Provide sufficient time for the student to think through the problem, before providing any kind of support to the child, as summative assessment is the objective of this exercise.
- Feedback is not to be given during the task; after the task is completed, the teacher can have a discussion with the whole class.

Rating Scale for Teacher Observation

Observation	Always	Sometimes	Never
Records the length and breadth of rectangles correctly.			
Calculates the area and perimeter for each rectangle.			
Observes that the area of the rectangle remains the same, even if the length and breadth change.			
Deduces the change in perimeter when rectangle becomes longer and thinner.			
Uses mathematical vocabulary related to area and perimeter to interpret a given context.			

Table 2

This strategy enables a formative use of summative assessment because the teacher uses the learner's responses to diagnose misconceptions and provide feedback, and learners reflect on their understanding of the relationship between area and perimeter, turning the end-of-unit task into a learning opportunity. A somewhat similar exploration may be done with sticks of the same length to make rectangles with fixed perimeter but varying area.

Here the comprehensive and multidimensional nature of summative assessment also plays an important role, using a variety of methods, such as teacher observations, written work, oral responses, and performance-based tasks, to capture the full range of student learning.

Strategy 2: Project

A project is a form of assessment which can connect Mathematics with real world applications. It provides the scope for formative assessment and at the same time, is summative in nature [1].

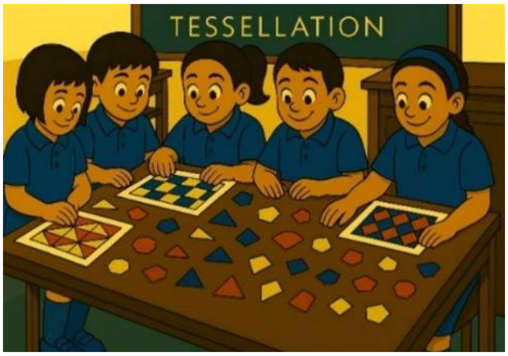


Figure 2: This image is generated using an AI tool

While doing a project, students work together in groups, with the teacher taking on the role of a guide and a facilitator. Time slots during school hours are set aside for them to plan, discuss ideas, and complete the work, with the teacher guiding them whenever needed. Students are encouraged to use low-cost or no-cost materials that are easily available. Projects are practical, hands-on, and possible for every student to do. They also help students learn how to work as a team and solve problems together.

Overview: This project is designed for Class 5 students. Students take on the role of pattern designers, diving into the fascinating world of tessellations. They start by exploring which polygons can tessellate and intuitively uncover the reasons behind it. This project will develop spatial reasoning through pattern making. Along the way, they experiment with combining different shapes and bring their creativity to life by designing theme-based tessellation art. A flow down of the learning standards with reference to NCF SE 2023 [4] for the project is given below:

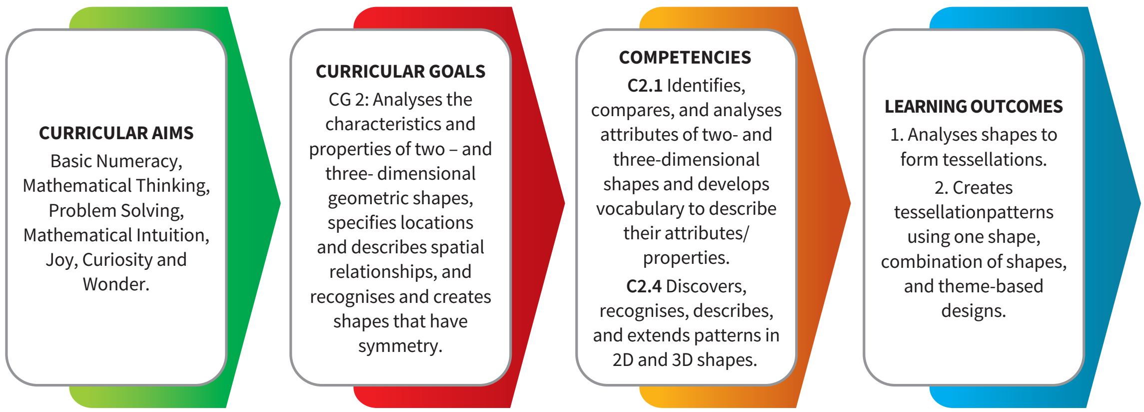


Figure 3

Group Task

The teacher provides a kit containing shapes to each group of three or four students. The kit contains cutouts (one each) of a regular triangle, square, regular pentagon, regular hexagon and regular octagon, all having the same side-length. The kit also has a rectangle whose breadth is equal to the side of the square and whose length is twice its breadth. There are three stages in this project. In each stage, students will make tessellations, on dot sheets, chart paper or any other creative mode. It is presumed that students are familiar with these shapes and have attempted tessellation activities. Students may replicate the shapes in the kit as per their need and can be creative using colours or patterns. The teacher can complete the assessment across the periods allocated in a week.

Stage 1 - Single Shape Tessellation (50-60 minutes)

- Classify the shapes in the kit based on whether they tessellate or not.
- Choose any two shapes that tessellate by themselves. Create single shape tessellations with each.

Stage 2 - Mixed Shape Tessellation (30-40 minutes)

- Identify all the pairs of shapes that tessellate with each other.
- Make a mixed shape tessellation using one of the identified pairs of shapes.
- Explain the pattern formed in your own words.

Stage 3 - Creative Theme Tessellation (80-90 minutes)

- Choose a theme from your surroundings (For example Nature, Cityscape or Weaving).
- Create tessellations forming designs for wallpaper, carpet, clothing or any other art form using at least two shapes, based on your theme.

Each group will display their tessellations and share their reflection with the whole class after the completion of all the three stages. Each group can be given 20 minutes to prepare their presentation and 8-10 minutes for their presentation.

There is an aspect of reflection included in the presentation. The teacher has to enable this by sharing some guiding questions with the class, while they prepare their presentations. Here are a few guiding questions which could be used:

1. How did you decide which shapes to use to create the design?
2. You might have faced some challenges during the process. Share an example of a challenge and how you resolved it.
3. What did you learn from working with the team?

Teachers can use rubrics to assess the projects. The main purpose of rubrics is to assess performances [2]. For some performances, you observe the student in the process of doing something, and for other performances, you observe the product that is the result of the student's work. Rubrics can be developed based on criteria such as *pattern accuracy, artistic execution, mathematical reasoning, presentation, and collaboration*, using a three-point scale. A sample given in Table 3, will help the teacher to assess the students (in small groups) during the three stages of the project. The total score is taken as 10. The indicators for this rubric have been derived from the learning outcomes of the project, listed in the flow down above (Figure 3).

Part of the Project	Score	Indicators*
Stage 1	3	→ Identifies three or more shapes that tessellate with itself → Identifies two shapes that do not tessellate with itself → Makes two single shape tessellations → Demonstrates good coordination during the process
	2	→ Identifies two shapes that tessellate with itself → Identifies one shape that does not tessellate with itself → Makes two single shape tessellations → Demonstrates good coordination during the process
	1	→ Identifies one shape that tessellates with itself → Does not identify any shape that does not tessellate with itself → Makes a tessellation using single shape → Struggles to coordinate with others
	0	→ Identifies no shape that tessellates with itself → Does not make a tessellation → Struggles to coordinate with others
Stage 2	3	→ Identifies more than two pairs of shapes that tessellate with one another → Makes a mixed tessellation with any one pair of shapes → Explains the pattern in their own words (during presentation) → Demonstrates good coordination during the process
	2	→ Identifies two pairs of shapes that tessellate with one another → Makes a mixed tessellation with any one pair of shapes → Struggles to explain the pattern (during presentation) → Demonstrates good coordination during the process

Part of the Project	Score	Indicators*
	1	→ Identifies one pair of shapes that tessellate with one another → Struggles to make a tessellation with the identified pair of shapes → Struggles to explain the pattern (during presentation) → Struggles to coordinate with each other
	0	→ Cannot identify a pair of shapes that tesselate with each other → No tessellation is made → Struggles to coordinate with each other
Stage 3	4	→ Creates a tessellation using at least two shapes representing the chosen theme → Orally presents a detailed analysis of the design in terms of shapes used for tessellation → Demonstrates good coordination during the designing process and presentation → All team members are reflective of the process
	3	→ Creates a tessellation using at least two shapes, representing the chosen theme → The oral analysis is not detailed; but it does cover some aspects of the design → Demonstrates good coordination during the designing process and presentation → Only some team members are reflective of the process
	2	→ Creates a tessellation using at least two shapes, but it does not represent the chosen theme → Does not have an analysis of the design → Some participants are active while others are passive in doing the work → Only some team members are reflective of the process
	1	→ Creates a tessellation using at least two shapes, but it does not represent the chosen theme → Does not have an analysis of the design → Lacks coordination in doing any task during the process → Reflections are not a part of the presentation
	0	→ Does not have a design to present

*These are some representative indicators. Teachers can develop rubrics with similar indicators at various levels.

Table 3

Note to the Teacher

- This rubric is to be used by the teacher to assess the students during the process of doing the project. The indicators will guide the teacher to note down the observations about each group.
- Refer to NCERT Class 5 chapter 7 (Shapes and Patterns) for more details on tessellations.
- Teacher may add other tools like checklist or rating scale to incorporate individual assessment of students.

This project, as summative assessment, clearly provides opportunities for students to synthesize their learning, tracks students' progress against learning standards and determines whether students are ready to progress to the next stage.

Strategy 3: Demonstration Task

Geometry chapters such as Fun with Symmetry, Shapes and Patterns around us, and Hide and Seek from the Class 4 NCERT Mathematics textbook [5] can be assessed through *demonstration* as well. In the proposed assessment, the teacher needs to spend quality time with the students in the classroom, math lab, or any available open space, so that children have enough space to spread out the materials and work with them. Each child is given these questions to work on individually to demonstrate their understanding. A few examples of tasks for demonstration to assess competencies like *'Identifies, compares, and analyses attributes of two- and three-dimensional shapes and develops vocabulary to describe their attributes/properties'* and *'Discovers, recognises, describes, and extends patterns in 2D and 3D shapes'* [4, P.274] are given below.

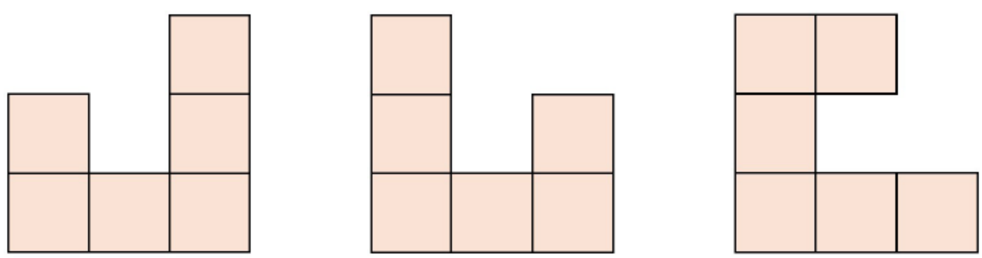
Materials needed (per child):

Task 1: Cubes or building blocks or match boxes (all of same size) - 10

Task 2: Pieces of straws / sticks of equal length - 9

Task 3: Cutouts from papers of two different colours - 30 (of each colour)

1. Imagine that you are viewing a building from different positions. Figure 4 shows views of the front, top and right side. Each pink square represents a cube. Using ten such cubes, create a model of the building.



Front View   Top View   Right side view

Figure 4

   (a) In how many ways could you make a building, using all the ten cubes? Demonstrate.

   (b) Suppose the given information is only front view and the top view, while the side view is not given. Does the building remain the same as in (a)? If not, show two or three different buildings you can make.

2. You are provided with straws / sticks of same length. Make three triangles, each having sides of the same length using exactly (a) 7 straws (b) 8 straws (c) 9 straws. (Remember you cannot bend or cut straws. They cannot be inserted into one another also.)

3. You are given a square grid sheet. You can fit in the two given tiles in each square of the grid. Try out different positions of the tiles (standing, lying down) and create designs for tiling walls or floors. In each design, you can find a repeating unit. Identify the repeating units. Come up with at least two such designs.

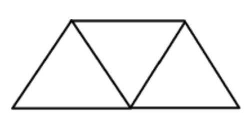
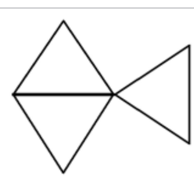
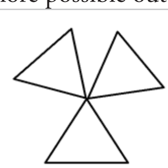
Note to the Teacher

- This task emphasizes the capacity of visualisation, which cannot be assessed in a pencil and paper test. More such tasks can be designed by the teacher.
- The teacher may decide to choose one or more tasks, depending upon the time decided by the school for this assessment and the availability of materials.
- 20–30 minutes can be allotted for task 1 and 10–15 minutes each for Tasks 2 and 3. Teachers can decide the time for each task based on their understanding of the class.

A checklist can help students assess the completeness of their work, so they know they are turning in what is required and are developing work habits in the process [2]. A sample of self-assessment checklist is given here. This helps the learners assess their own achievement.

What to look for?	Yes / No
Task 1	
• I am able to place the blocks with respect to each view - front, top, side.	
• I am able to imagine / visualize the building using the given views.	
• I could identify that if the side view is not given, there could be many possible buildings.	
Task 2*	
• I can create three triangles with 7 straws.	
• I can create three triangles with 8 straws.	
• I can create three triangles with 9 straws.	
Task 3	
• I was able to create two designs using the two tiles in different ways.	
• I could identify the repeating unit of each design I made.	

*Some possible outcomes for Task 2 are given below:

Number of Straws	Some Possible outcomes with three triangles
7	
8	 Note: There are more possible outcomes in this case.
9	 Note: There are more possible outcomes in this case.

The teachers can also design a rubric to assess the demonstration.

It is hoped that the three strategies described above will inspire teachers to go beyond pen and paper tests, especially for summative assessment. While the tasks may be intimidating initially, each attempt will give the teacher more ideas for improvement in their students' context and ability. It is expected that children are given ample opportunities to work with materials during concept development, before providing such tasks for summative assessments. With tools like rubrics and checklists which compare the performance of a child against the prescribed competencies, teachers can easily provide qualitative descriptions. By reflecting on the uses of summative assessment, the transitions that the child undergoes in the preparatory stage and by referring to the prescribed competencies, teachers can design assessment tasks that focus on relaxing the student rather than building on their fear and tension. They can be given a sporting chance to help them perform well in high-stakes assessment, not just for marks but also to meet challenges which develop their cognitive abilities and affective skills.

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The Tricky Truth about Visualising Fractions

Kanchana Suryakumar

All is not as it seems!

I have often seen my students, even in high school, make mistakes in fraction addition such as the one shown in Figure 1. I routinely attributed this to an error in understanding the underlying algorithm, what I refer to as a "method error". I did not see this as an issue with conceptual understanding, until I was unexpectedly pulled into a primary classroom one day.

$$\frac{4}{4} + \frac{1}{4} = \frac{5}{8}$$

Figure 1: A common error in fraction addition

A substitute teacher caught me passing by her class and called me in to help her. She had the diagram in Figure 2 drawn on the board. Before we get to the reason the teacher invited me into her class, take a moment to think about the question in the figure. What do you think the answer to the question is? Can you explain the reasoning behind your answer?

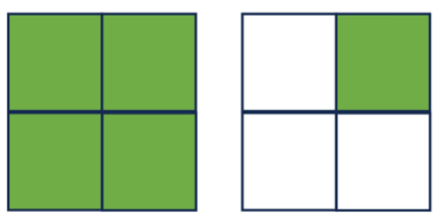


Figure 2: What fraction of the figure is shaded?

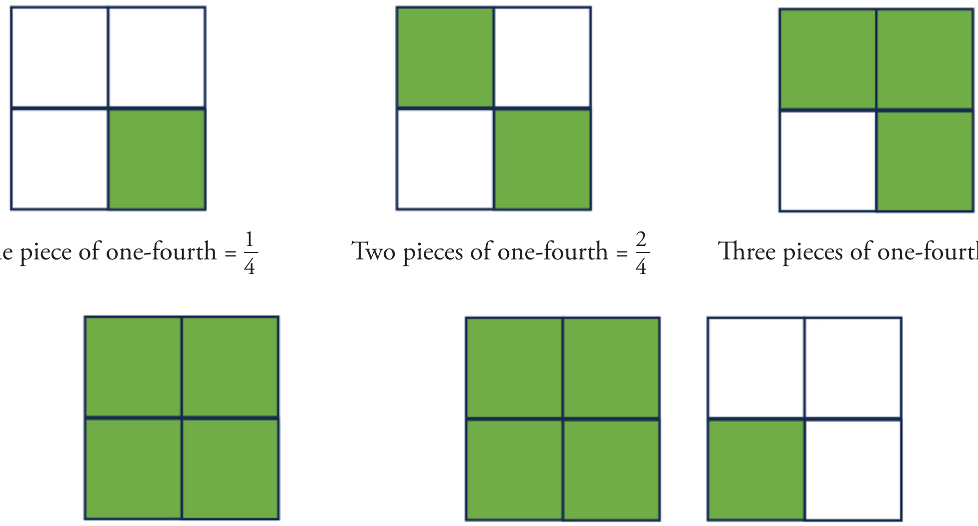
The students in the classroom had come up with two answers for the question. The teacher requested, "Can you explain to the children why the shaded region represents $\frac{5}{4}$ and not $\frac{5}{8}$?"

How would *you* respond to this request?

The burden of an unsatisfying solution

I have to admit that I was thrown off-guard. While I understood how method-based errors could creep up in student work, I was at a loss to understand how even visual diagrams could lead to misunderstanding. I would like to believe that I recovered quickly enough and convinced at least a few students why the shaded region represented $\frac{5}{4}$. You might be familiar with the argument that I presented. As illustrated in Figure 3, I started with one "cake" cut into four equal parts and tried to build up the idea of non-unit fractions as multiple slices/pieces of a unit fraction (one-fourth in this case). Two pieces of one-fourth would be numerically represented as $\frac{2}{4}$ and be read as two-fourths, and so on up to five-fourths, which is 1 whole cake and a one-fourth.

Keywords: fractions, whole, visualising fractions, misconceptions



One piece of one-fourth = $\frac{1}{4}$ Two pieces of one-fourth = $\frac{2}{4}$ Three pieces of one-fourth = $\frac{3}{4}$

Four pieces of one-fourth = $\frac{4}{4}$ = 1 whole cake

Five pieces of one-fourth = $\frac{5}{4}$ = 1 whole cake and a one-fourth

Figure 3: Illustrating the meaning of non-unit fractions

The confusion from Figure 2 is a common one that we witness when fractions are taught as "number of shaded parts ÷ total number of parts". So, instead, I tried to draw the students' attention to the size of each piece and what a "quarter" or "one-fourth" meant.

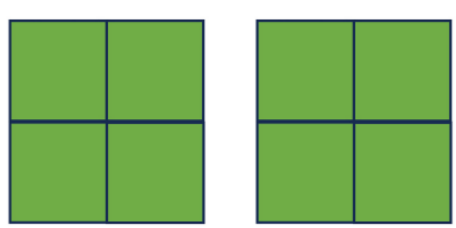
Apart from some uncertainty over whether I had pitched my reasoning at the right level for primary school, I also left with a nagging feeling that the confusion was perfectly valid. Visualising fractions was not as intuitive as I thought it was!

The light at the end of the tunnel

It was not until a year or two later that I ran into an explanation that made *most* sense to me. The lecturer at a teacher professional development course that I was attending, posed the question from Figure 2 and asked if the shaded region was $\frac{5}{4}$, $\frac{5}{8}$, or $\frac{5}{2}$. Imagine my confusion at seeing $\frac{5}{2}$ as an option!

Let me spare you the misery, and cut to the chase. The key to making sense of this question is to ask **"what is one whole?"**

If one whole is



then, the fraction that captures the shaded region in Figure 2 is $\frac{5}{8}$.

Can you now see how $\frac{5}{2}$ could possibly be an answer?

If one whole is

then, the fraction that captures the shaded region in Figure 2 is $2\frac{1}{2}$ or $\frac{5}{2}$.

When I jumped into my explanation in the primary classroom, I did so with my own assumption of one whole - absolutely clear in my mind that there could be no other logical reasoning. It is important that I explicitly state the value of asking "what is one whole" and understanding the multiple solutions for this seemingly harmless problem. This kind of understanding and reasoning can help a teacher differentiate between an error and a misconception, and provide appropriate clarification and support to the student. I do not mean to suggest that a teacher should provide such a detailed explanation every time the classroom stumbles upon improper fractions. Just repeatedly drawing attention to "what is one whole?" may suffice in most cases.

Into the light

Through the numerical example in Figure 1 and its visual equivalent in Figure 2, I have tried to draw attention to a common confusion in fraction addition. Based on a personal anecdote, I make the case that one possible explanation for this confusion could be our understanding of what "one whole" is. I argue that explicitly stating or asking what "one whole" looks like can help with better conceptual understanding and challenge long-held assumptions.

The accompanying worksheet showcases this idea and provides multiple examples for you to try and reflect on.

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Worksheet
(Based on the Tricky Truth about Visualising Fractions)

Kshama Chakravarthy

Here is a worksheet that teachers can use to check their students' understanding of fractions, in the context of the accompanying article. It may be used for students in classes 4, 5, 6 and 7 as appropriate, to assess their understanding. The pre-requisite is the idea of a whole and how the fraction changes depending on the whole that is considered. This is to be explained to students before they attempt the worksheet so that students don't feel overwhelmed by the questions.

1. Draw your own "whole" and shade $\frac{1}{4}$ of it.

2. Draw a whole such that:-

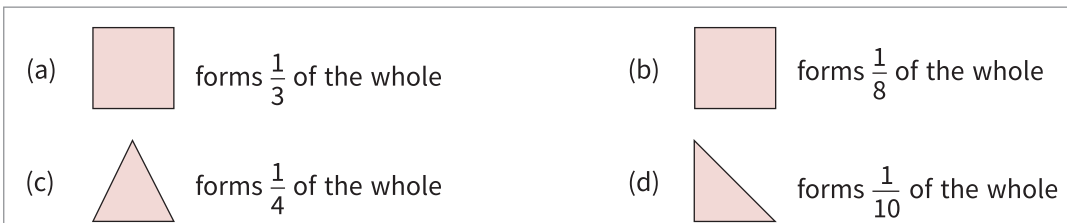


Figure 1

3. Read the statements below.

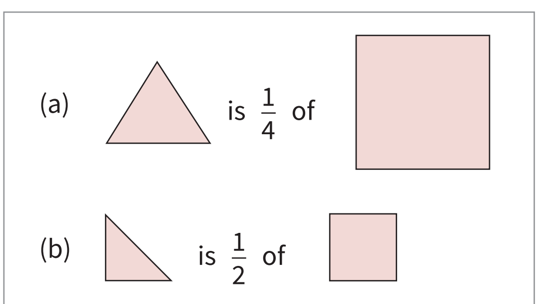


Figure 2

Does this mean that $\frac{1}{4}$ is larger than $\frac{1}{2}$? Explain the situation/Justify your answer.

Worksheet

4. The shaded portion shown in Figure 3 represents $\frac{7}{6}$ of a whole; choose the whole from the options in Figure 4.

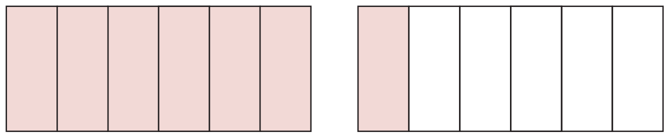


Figure 3

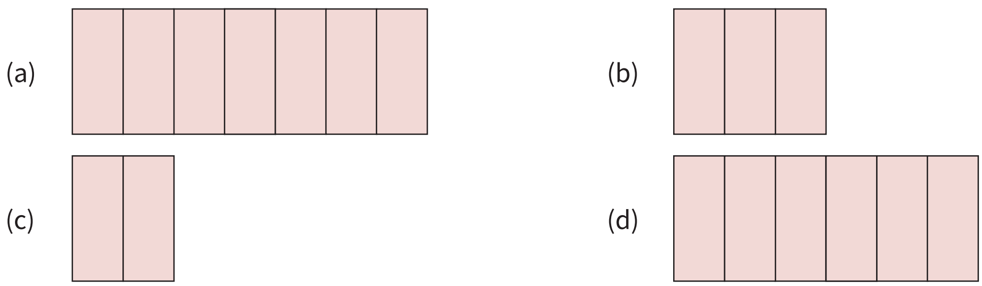


Figure 4

5. Look at Figure 5 and answer the questions that follow, with respect to this image.

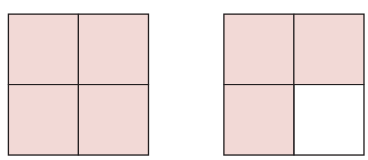


Figure 5

(a) If the shaded portion shown in Figure 5 represents $\frac{7}{2}$ of a whole, choose the correct option for the whole from Figure 6.

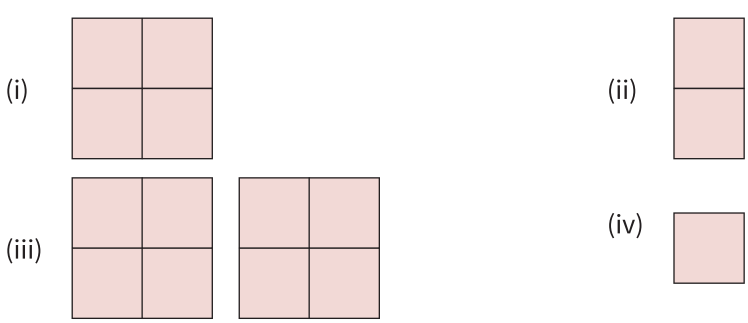


Figure 6

(b) If the shaded portion in Figure 5 represents $\frac{7}{8}$ of a whole, choose the correct option for the whole from Figure 7.

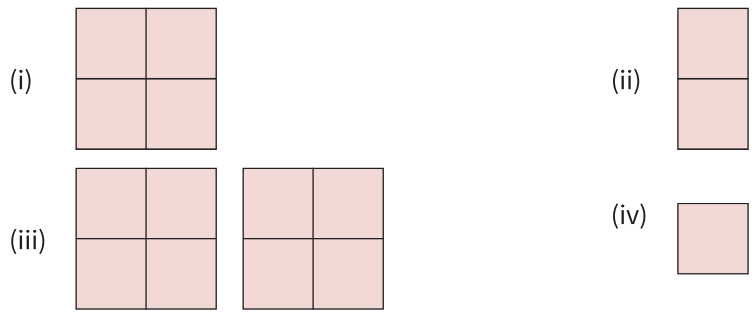


Figure 7

(c) If the shaded portion in Figure 5 represents $\frac{7}{4}$ of a whole; choose the correct option for the whole from Figure 8.

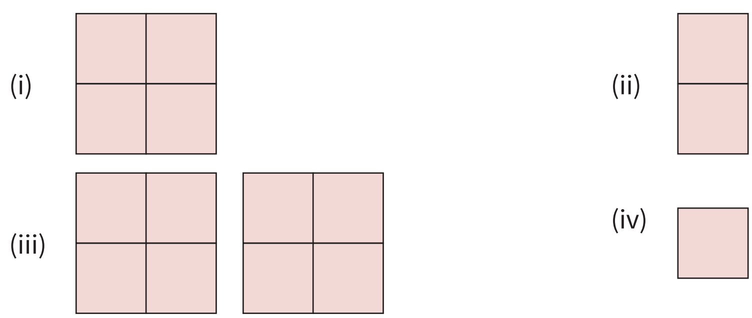


Figure 8

(d) If the shaded portion in Figure 5 represents 7 of a whole, choose the correct option for the whole from Figure 9.

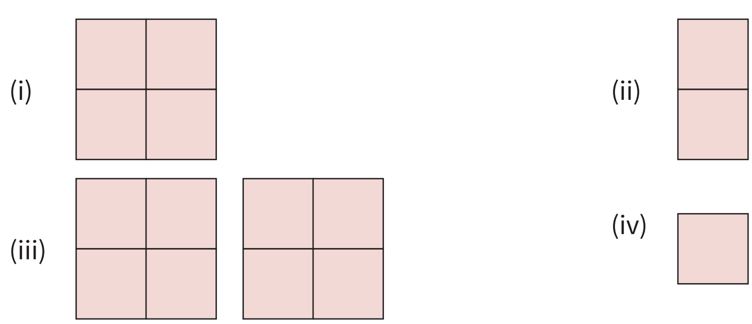


Figure 9

Worksheet

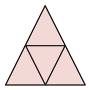
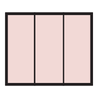
6. If  is a whole which of the following represents $\frac{3}{4}$?

Figure 10

7. If  is a whole which of the following represents $\frac{5}{3}$? Note: Tick as many correct options as you see.

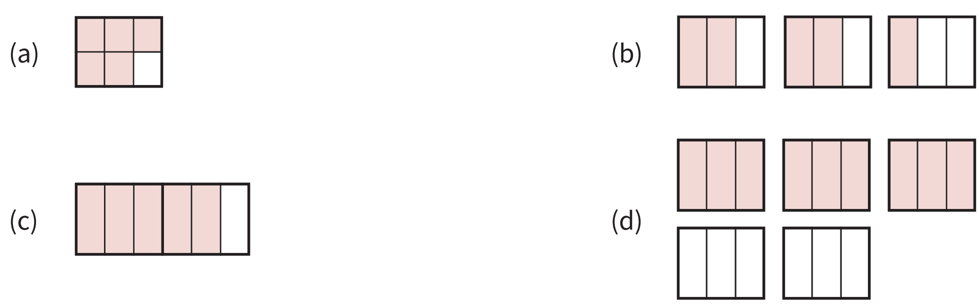


Figure 11

Teacher Notes

Q1: This allows students to be creative and make their own whole, and thereby also see for themselves that it is easier to divide regular shapes into fractions versus irregular shapes.

Q2: This question tests their understanding of fractions, and may also help them see that the shape of the whole can be different even though the fractional unit is the same. For example, in question (a), the whole can look like a horizontal or vertical row of 3 squares, or an L-shaped whole.

Q3: The importance of a whole comes out beautifully through this question. When comparing two fractions it is assumed that the fractions being referred to are of the same whole, but when a contradictory statement is made, it forces one to think when this can be true. Of course in this question, the image helps the student arrive at the answer. If the wholes being compared are different, the value of the fraction alters.

Q4: Questions usually give the whole and ask for the fraction that the shaded part represents. There is a twist here where the fraction is given and the student has to identify the whole. A good understanding of fractions will enable the student to work backwards to arrive at the answer.

Q5: This question really tests the understanding of fractions when the whole is different each time. The figure is the same, the shaded portion is the same, and yet, with a different consideration of the whole each time, the fraction that the shaded portion represents changes.

Q6: This is a standard/ regular question, with the whole being made explicit so as to not be ambiguous.

Q7: This question is slightly more challenging because it involves an improper fraction to be identified. A student who is unsure about the answer can be confused looking at the options. It requires them to identify a third of the whole and then look at 5-thirds.

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Turning the Hands of Time: A Classroom Journey with Half and Quarter Hours

Garima Bhatt

This article was written by Garima Bhatt, who saw models of some clocks based on the Time pullout from the March 2017 issue of At Right Angles (https://publications.azimpremjiuniversity.edu.in/1381/1/19_Teaching%20Time.pdf).

During a workshop in Bengaluru, I first encountered a simple yet ingenious Teaching Learning Material (TLM)— a *time clock*. I was immediately captivated. The idea was so clever and straightforward that I thought, "Why didn't we think of this?" I knew I wanted to bring it back to my classroom to help my Grade 3–5 students truly *see* time, not just read it.

Even though children may read off the time from a digital display, being able to read an analog clock remains very important. I've noticed instances where the child can read the time on a phone which has a digital display but gets completely confused by the analog clock in school. I realised that this affected their sense of time, planning, and daily routines. Analog clocks are valuable because they help children understand **fractions, elapsed time, angles and number patterns** in a hands-on and visual manner. And since schools, examination halls, and public places still use analog clocks, being confident about reading them is a practical life skill. That's why I prefer reinforcing the skill of reading an analog clock.

Inspired by the original clocks (shown in Figure 1), I designed my own version with a few innovations.

Figure 1

- I coloured it brightly to make it visually engaging — I chose **green for the quarter past clocks and red for the half past clocks** — and I marked all the minutes neatly around the clock to help students who struggled to remember how many minutes each number represents.
- I made **three different clocks:** one showing **half past** (Figure 2), one showing **quarter past** (Figure 3), and one showing **quarter to** (Figure 4) the hour, with the minute hand fixed but the hour hand fully movable. Concepts such as 'half past', 'quarter past' and 'quarter to' had always seemed abstract on paper, but this tool promised a way to make them tangible.

Keywords: time, intervals, clocks, analog, fractions, angles, number patterns, TLM

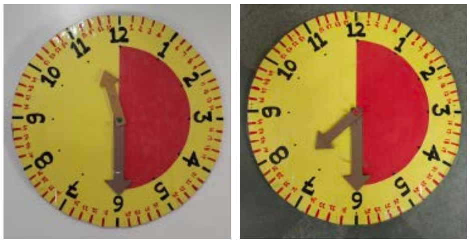

Figure 2

Figure 3

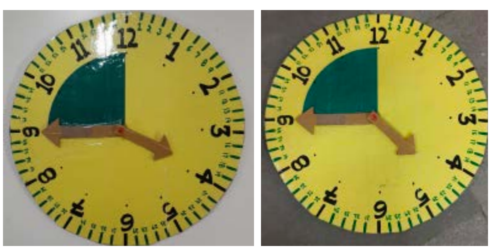

Figure 4

- To ensure durability, I laminated the clocks so they could be used repeatedly.

The class began by revisiting what they already knew. Students could identify the minute and hour hands of a clock and read simple times like 5:20 (20 minutes past 5) and 5:30. They had practised this using both oral and written exercises. During one such drill, a student shared the time in Hindi: “1:30 is *dedh*, 5:15 is *sava panch* — my mother says it like that at home.” I explained that in English, these correspond to ‘half past’ and ‘quarter past’, connecting prior knowledge to new learning. Since students were familiar with fractions, it was easy to explain that 15 minutes is a quarter of an hour, and 30 minutes is half an hour.

When I introduced the clocks in class, the students were immediately intrigued. Their first observation was that the big hand was glued in place. “Ma’am, why is the minute hand fixed?” they asked. Before I could answer, one student suggested, “So we can focus on the hour hand!” Another noticed, “Wow, that’s clever! Now we can see all half past hours clearly.” Suddenly, the language used became meaningful and concrete.

As they explored, one curious student asked, “Ma’am, does the hour hand go past the number a little bit, or does it have to stop exactly on it?” I explained that the hour hand doesn’t always sit exactly on a number — it depends on the time we are showing. For example:

- For **half past 1 (1:30)**, the hour hand is **exactly halfway between 1 and 2**.
- For **quarter past 2** (2:15), the hour hand is **a little past 2, closer to 2 than 3**.
- For **quarter to 5** (4:45), the hour hand is **a little before 5, closer to 5 than 4**.

Then students started to place the hour hand exactly where it needs to be. This distinction helped students visualize the gradual passage of time and understand the subtle positioning of the hour hand for each case.

The following interaction took place in Class 4. There were 28 students, so I created groups of four students each. Each group received one of the three clocks — half past, quarter past, or quarter to — and explored how to position the hour hand correctly for any hour. I ensured that each group had the opportunity to work with all three clocks. They experimented with placing it **exactly between two numbers for half past and closer to the relevant hour for quarter past or quarter to**. Students rotated the hour hand repeatedly, comparing their observations with the Hindi terms they knew and discovering the logic of English time terminology for themselves. They excitedly predicted times, experimented with moving the hour hand to different positions, and discussed amongst themselves: “If the hour hand moves, it tells us exactly how far into the hour we are!” Some asked if they could try making other clocks showing times like 2:40 or 6:10 or 11o’ clock, by fixing the minute hand at 8, 2 and 12, respectively. They weren’t memorizing rules — they were reasoning, experimenting, and learning through discovery.

Reflecting on this experience, I realized that the TLM not only reinforced time-related terminology but also kept the concepts alive because I revisited it periodically, even after moving on to other topics. A short question here or a minute of play there reminded students of half past, quarter past and quarter to. They remembered because they had interacted with the tool joyfully and repeatedly. Even shy students participated enthusiastically, encouraged by the hands-on exploration.

This simple, colourful time pullout clock transformed abstract notions into visible, tangible experiences. It allowed students to take ownership of their learning, explore independently, and understand the subtle movement of the hour hand in relation to fractions of an hour. For me, it reinforced a vital lesson: thoughtfully designed TLMs can turn challenging concepts into joyful discoveries, making the classroom a place where students truly experience learning.

Editor's Note:

1. Teachers can build the fraction connected to (i) *'saade'*, (ii) *'sava'*, (iii) *'poune'*: These 3 words in Hindi and some other Indian languages refer to
    - Half past, or half more than – e.g., *saade char* (four) = $4\frac{1}{2}$ or half past 4, i.e., 4:30
    - Quarter past, or quarter more than – e.g., *sava sat* (seven) = $7\frac{1}{4}$ or quarter past 7, i.e., 7:15
    - Quarter to, or quarter less than – e.g., *poune aat* (eight) = $8 - \frac{1}{4} = 7\frac{3}{4}$ or quarter to 8, i.e., 7:45
2. Why do we say 'half past', 'quarter past', but 'quarter to'? This was asked by a bunch of primary school children from Pokhrama, Bihar. Half past and quarter past indicate that we are adding more to what is already there. But 'quarter to' indicates that we are a quarter unit away from reaching the given number. For example, quarter to eight means quarter of hour, i.e., 15 minutes before 8, i.e., 7:45. The phrase 'quarter to 8' is equivalent to 'three quarter past 7' or 7:45. But that is a longer phrase. So, we have chosen to use the former. Similarly, 'quarter past 6' is equivalent to 'three quarter to 7' or 6:15.
3. A question for the reader: Do you notice the difference between the quarter to clock in the Time pullout (Figure 5) vs Garima's version in Figure 4? Which one do you prefer and why? Please let us know at AtRightAngles.editors@apu.edu.in

Figure 5

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Garima currently focuses on mathematics education in the primary grades, with a strong belief in making math fun, engaging, and meaningful for young learners. She enjoys creating a classroom environment where children can explore mathematical ideas through hands-on activities, games, and real-life connections, helping them build confidence and curiosity. Garima may be contacted at garima.bhatt@azimpremjifoundation.org

Thinking out of the Box - A Worksheet

Swati Sircar

Monika was teaching 'Nets of Cuboids' with Class 5. During the reflection and discussion after the sessions, the mathematics teacher pointed out two misconceptions that arose during her sessions. The cuboid that she chose included two square faces. Some of the students formed the misconception that the net of a cuboid must have at least one pair of square faces. The worksheet given below is intended to address this misconception.

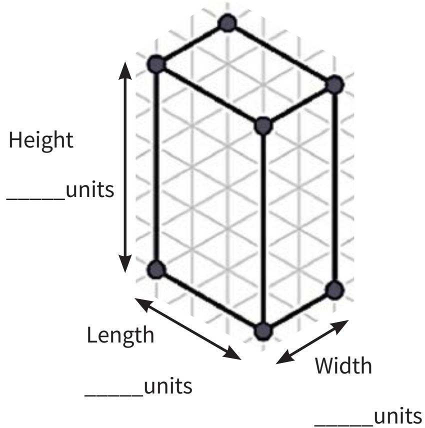
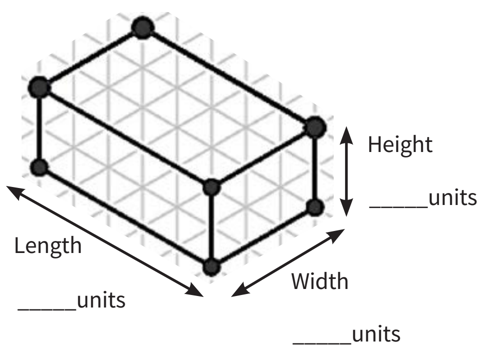
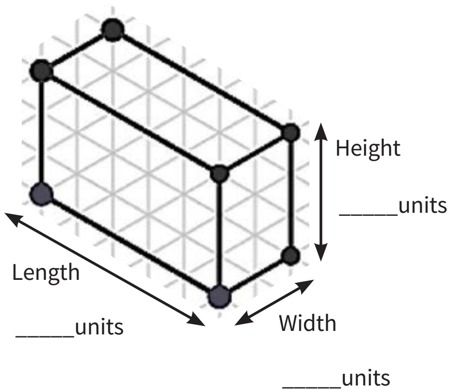
As per the new Class 3 NCERT textbooks, cubes and cuboids are introduced in Chapter 2 and it is expected that students make different solids by combining several (identical) cubes. Edges and corners are also introduced. Squares and rectangles are introduced in Chapter 5. In Chapter 1 of the Class 4 textbook, nets of various solids including cubes and cuboids are discussed, and children learn that the net of a polyhedron is made of its faces. Using triangular dot paper, they draw sketches of cubes and cuboids. In Chapter 2, they understand perspective view. Therefore, this worksheet can be used in Class 4 or in Class 5.

Keywords: cuboids, cubes, faces, edges, nets, dimensions

Worksheet

Name: ____

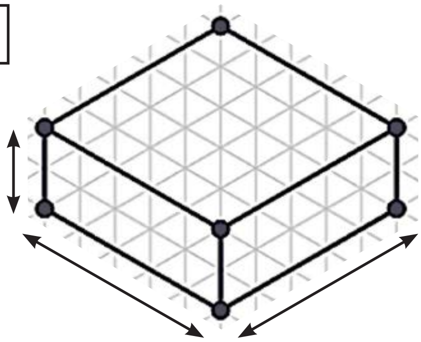
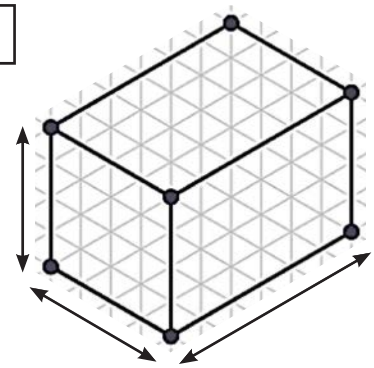
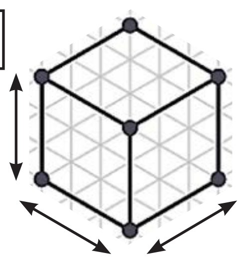
1. Use the grid to find the dimensions of the cuboids given below:

(a) 	**a. This cuboid is** ________ units by ________ units by ________ units Therefore, its faces are (i) ________ units by ________ units (ii) ________ units by ________ units (iii) ________ units by ________ units
(b) 	**b. This cuboid is** ________ units by ________ units by ________ units Therefore, its faces are (i) ________ units by ________ units (ii) ________ units by ________ units (iii) ________ units by ________ units
(c) 	**c. This cuboid is** ________ units by ________ units by ________ units Therefore, its faces are (i) ________ units by ________ units (ii) ________ units by ________ units (iii) ________ units by ________ units

Are these cuboids the same or different? Give reasons for your answer.

Worksheet

2. Now find the dimensions of these cuboids. If any of the cuboids have square faces, write down the number of square faces in the box at the top left.

(a) 	**a. This cuboid is** ________ units by ________ units by ________ units Therefore, its faces are (i) ________ units by ________ units (ii) ________ units by ________ units (iii) ________ units by ________ units
(b) 	**b. This cuboid is** ________ units by ________ units by ________ units Therefore, its faces are (i) ________ units by ________ units (ii) ________ units by ________ units (iii) ________ units by ________ units
(c) 	**c. This cuboid is** ________ units by ________ units by ________ units Therefore, its faces are (i) ________ units by ________ units (ii) ________ units by ________ units (iii) ________ units by ________ units

3. Choose the right answer: The number of square faces that a cuboid can have is:

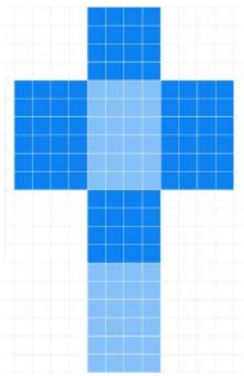
(a) 2, 4 or 6 (b) 0, 3 or 6 (c) 0, 2 or 6 (d) 0, 2, 4 or 6

4. If a cuboid has at least one pair of square faces, what do you notice about the remaining faces?

Worksheet

5. Can you match the nets below with the cuboids in Question 2.

Hint: Find the dimensions – length, width, height – of each one.



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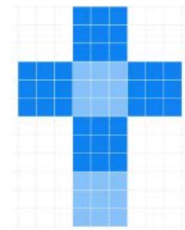

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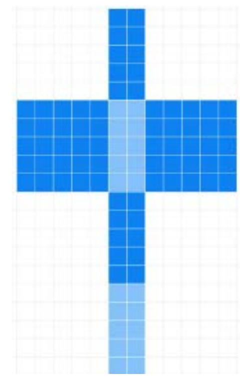

____

6. In the grid below, draw the net of any cuboid from Question 1.

Worksheet

7. Pick any 3 numbers (each less than 10). ____________, ____________, ____________

a. Now consider a cuboid with these dimensions, that is, ____ units by ____ units by ____ units

b. Draw a rough sketch of the cuboid and label its dimensions.

c. Draw the net of this cuboid.

Teacher Notes

This worksheet is inspired by misconceptions that surfaced during the practice teaching internship of Monika Mahaldar, which she did at Azim Premji School, Dhamtari. Monika is a 4th year BSc-BEd (Mathematics) student at Azim Premji University, Bengaluru.

Teachers may identify such misconceptions by asking each student to list at least two daily items that are cuboids with no square faces (phone, pencil box, TV remote, books, etc.).

By exploring the dimensions of different cuboids and viewing them from different perspectives, students arrive at the understanding that a cuboid may or may not have square faces. The teacher could start Question 2 by asking students to identify if all the given solids are cuboids. The questions help them to introspect on the possible number of square faces that a cuboid can have. They may also notice the implication that having one pair of square faces will have on the remaining faces. They move from the cuboid to its net and then from the net to the cuboid. By this exploration, the original misconception provides rich pickings for the introspecting teacher and the curious student.

A second misconception that surfaced is worth mentioning, though not addressed in this article. Sometimes we use squares made of thick material, such as 4mm ethyl vinyl acetate (EVA), or wooden squares with measurable thickness. To children, these are not 2D but 3D, and it reflects in how they use or engage with similar materials. They go beyond the intended use perceived by the adults. So, it is a better idea to use thin material, such as card, with negligible thickness, for squares, rectangles and other 2D shapes.

SWATI SIRCAR is a consultant with Azim Premji University. Mathematics is the second love of her life (the first being drawing). She is a B.Stat-M.Stat from Indian Statistical Institute and an MS in math from University of Washington, Seattle. She has been doing mathematics with children and teachers for more than 15 years and is deeply interested in anything hands on - origami in particular. She initiated Math Space, a mathematics laboratory at the Bengaluru campus of Azim Premji University. It caters to schools, teachers, parents, children, NGOs working in school education and teacher educators. Swati may be contacted at swati.sircar@apu.edu.in

SUMS OF ODD AND EVEN NUMBERS

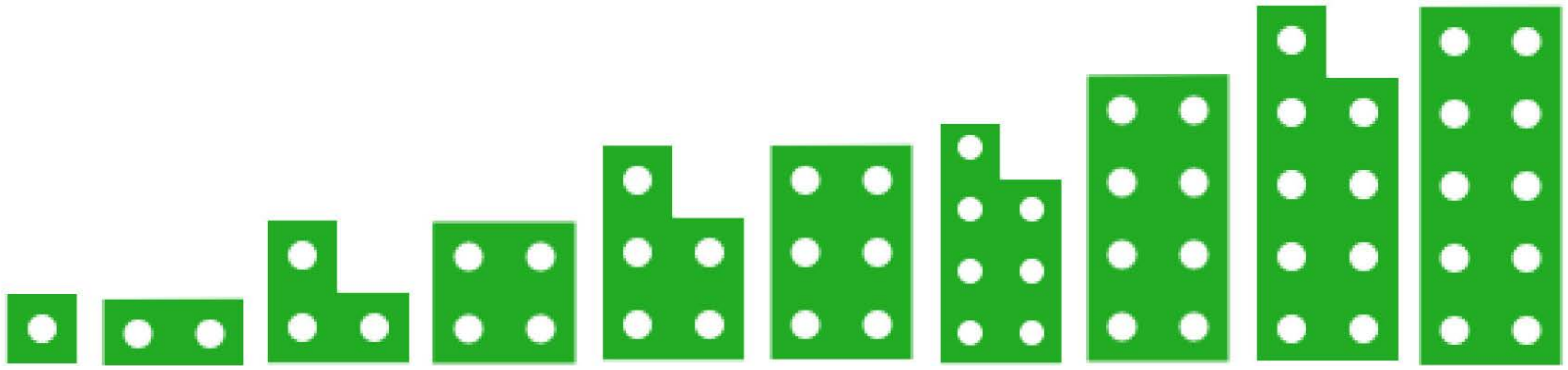

Figure 1: These are the first ten natural numbers, 1, 2, … 10 shown as ten-frame cutouts. The last one (10) is called a full frame. What do you notice about the tops of alternate numbers?

In each of the images in the three groups A, B and C, two numbers (shown in contrasting colours) are added. What do you notice about these numbers and about their sum?

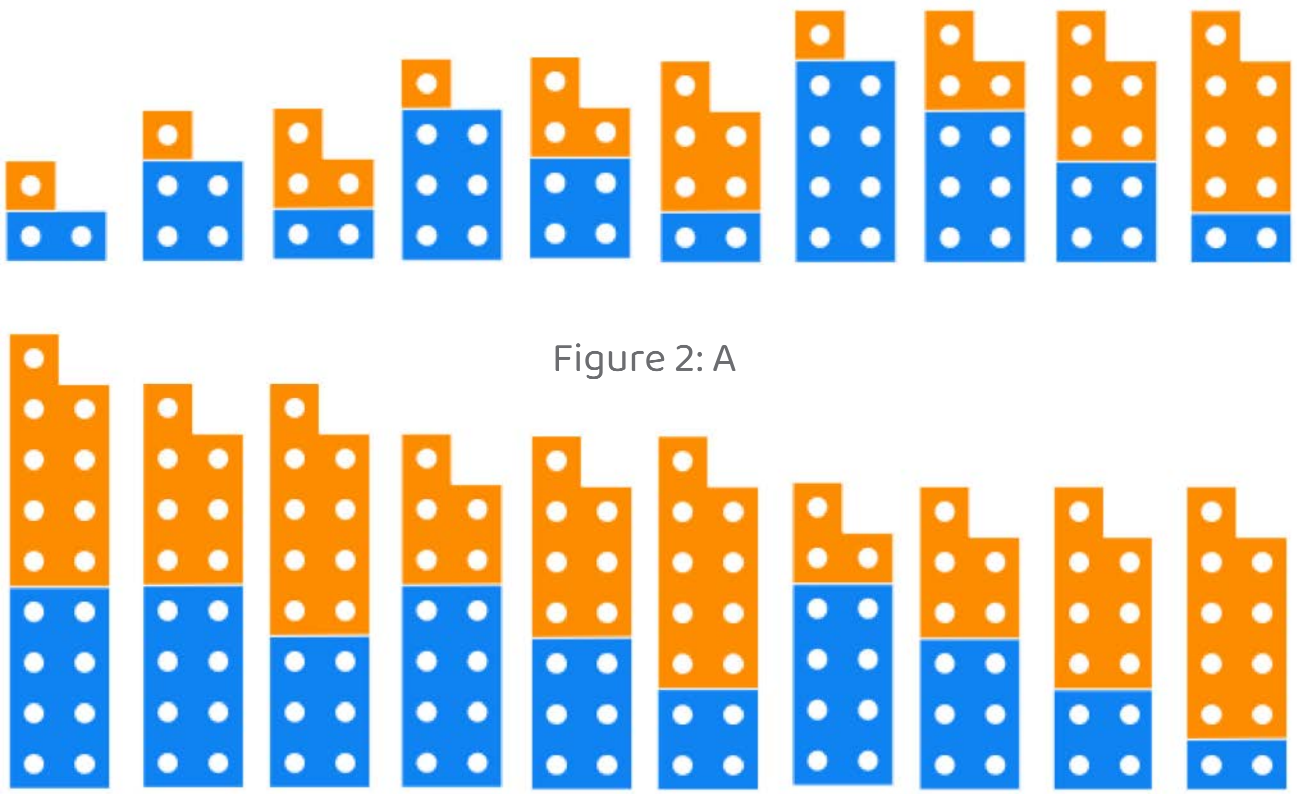

Figure 2: A

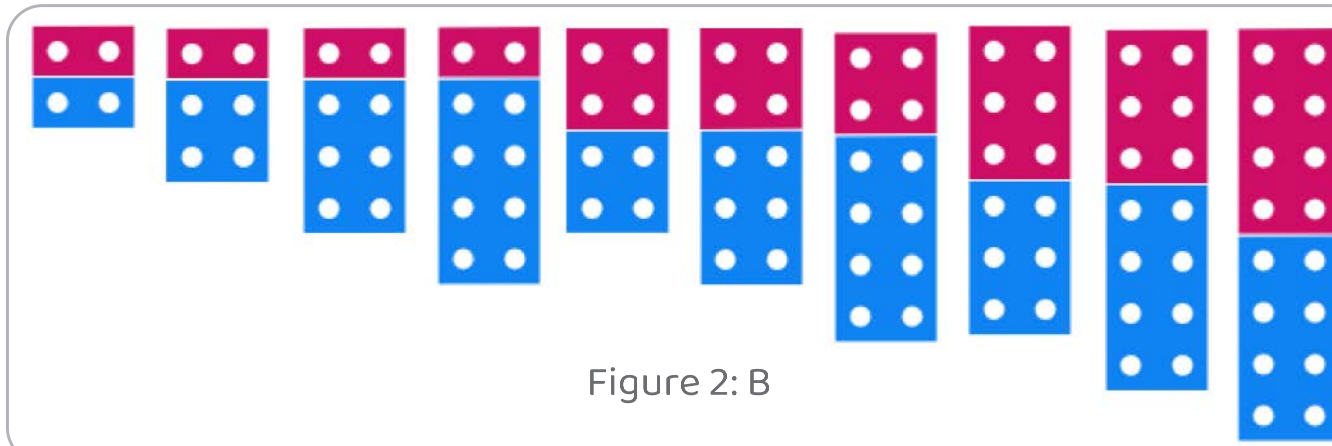

Figure 2: B

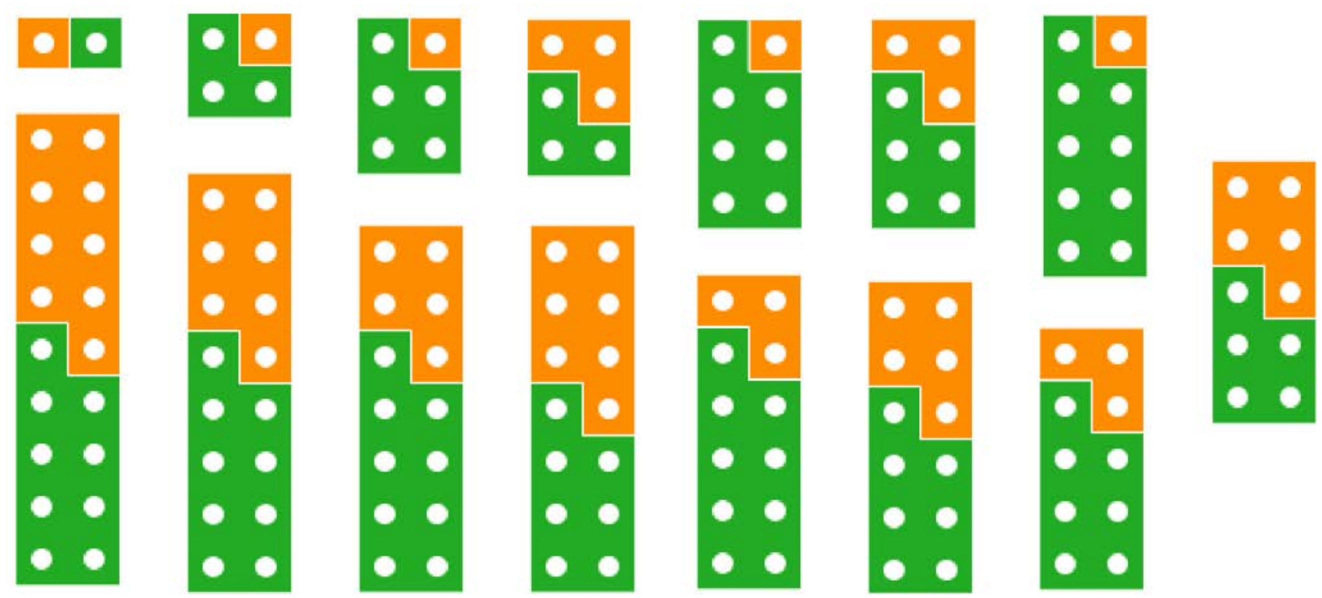

Figure 2: C

Can you make 13, 18, or any whole number greater than zero if you have enough ten-frames? How?

What about 125? 602? 1234?

Why does the units digit of a number determine whether it is odd or even?

Teacher Notes and suggested questions are given in an accompanying page

Using Ten-Frames to Find the Sums of Even and Odd Numbers

Math Space

Ten-frames are 2 × 5 frames that can be used to show the first ten natural numbers 1, 2, 3, … 10. You can find more details including how to make these at https://bit.ly/4kmFdoh

The poster *Sums of Even and Odd Numbers* is intended to be displayed in class for students to move from the concrete model of the ten-frame to representations of selected combinations of numbers from it. Students should be given adequate time to observe patterns, think about the questions posed and discuss their thoughts with their peers. Even a simple listing of the addition problems shown helps them to realise how many such combinations are possible and if all such combinations have been shown. From simple observation to generalisation is a powerful step in mathematics and this poster scaffolds such learning. The teacher notes given below are intended to help the teacher facilitate such a journey and it should be noted that simply focusing on the answers will not help the student develop their mathematical thinking skills. The questions are designed to provide the starting points, and the peer discussion will enable student-led discovery.

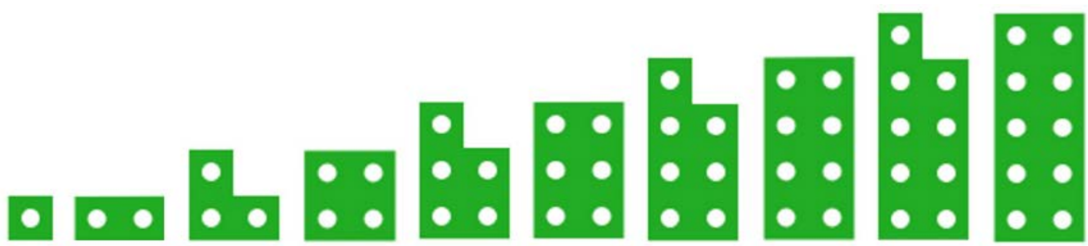

Figure 1

A careful observation using the trigger of the first question *What do you notice about the tops of alternate numbers?* in the poster shows that every alternate number beginning at the first, has a single 'odd' dot at the top, while the second, fourth, etc., numbers have a flat or 'even' top. This visualisation can help in connecting the 'parity of a number,' i.e., whether the number is 'odd' or 'even', and the mathematical concepts associated with these terms. Observe that this visualisation provides meaning to the standard algebraic approach of representing an even number as $2n$ and an odd number as $2m - 1$ or $2m + 1$ at a later stage.

Our answers for the second question *In each of the images in the three groups A, B and C, two numbers (shown in contrasting colours) are added. What do you notice about these numbers and about their sum?* are given below:

In Group A, we see the sums of all possible combinations of odd (orange) and even (blue) single-digit numbers.

In Group B, we see the sums of all possible combinations of two even single-digit numbers.

Similarly in Group C, we see the sums of two odd single-digit numbers.

Listing all the combinations of digits and observing their sums will help students work systematically.

Keywords: ten-frame, odd-even, parity, observation, conjecture

Teacher Notes

Questions that may be posed:

In Group A	In Group B	In Group C
The top number is ___________ (because ___________) and the bottom one is ______________ (because _________________) So, the sum is ______________ (because _________________)	Both numbers are ___________ (because _________________) So, the sum is ______________ (because _________________)	Both numbers are ___________ (because _________________) But, the sum is _____________ (because _________________)

- What is the smallest total in each group? And the largest?
- What is common to the sums of the numbers in each group? Can we generalise?
  - (i) Odd number + even number = _____ number
  - (ii) Even number + even number = _____ number
  - (iii) Odd number + odd number = _____ number

These visualisations help one understand how adding an even number doesn't change the parity of the sum from that of the number added to, i.e., odd + **even** = odd (as illustrated by A) and even + **even** = even (as depicted by B).

But adding an odd number changes the parity. When two odd numbers are added, they compensate for their oddness and make the sum even; the two odd ones sum to an even two as shown in C. In other words, **odd** + odd = even while B illustrates **odd** + even = odd.

Extension Question: Is zero odd or even?

There are multiple reasons why zero can't be odd. For example, if it is odd, then it needs to have an 'odd' dot at the top. But zero can't have any dot. So, it can't be odd. Therefore, it has to be even.

Also, adding an even number does not change the parity. So, odd + even = odd and even + even = even. Zero preserves this since a number does not change when zero is added to it. Therefore, zero must be even.

The next set of questions:

- *Can you make 13, 18, or any whole number greater than zero if you have enough ten-frames? How?*
- *What about 125? 602? 1234?*
- *Why does the units digit of a number determine whether it is odd or even?*

are again a step in the direction of generalisation.

Any whole number ≥ 10 is a bundle of tens and a 1-digit number (0, 1, 2, … 9). For example, 43 = 4 × 10 + 3, 602 = 60 × 10 + 2, 1234 = 123 × 10 + 4. So, each of these numbers can be represented with as many full-frames and then the 1-digit number, e.g., 43 can be represented as in Figure 2.

Figure 2

From this understanding, the generalisation that the units digit of a number determines its parity follows immediately.

If you find this interesting, check the review of ten-frames in the Nov 2022 issue of this magazine at https://bit.ly/4r79Qkb. It includes a worksheet.

Exploring Polyominoes and Nets of a Cube: A Classroom Reflection with Class 4 Students

Asma Memon

This article presents the responses and reasoning of Class 4 students from Shikha Academy, a low-income school in Mumbai, India that follows the Cambridge curriculum. The session involved 18 students and was based on a hands-on mathematical activity titled *"Polyominoes and Nets of a Cube"*, adapted from a worksheet published in *At Right Angles* by Azim Premji University (2022) (see https://bit.ly/4a5ztvB) **Polyominoes are plane geometric figures formed by joining two or more equal-sized squares edge to edge**, and they can take various shapes—such as dominoes, trominoes, tetrominoes, and pentominoes—depending on the number and arrangement of the squares.

Description of the exploration

The objective of the activity was to introduce students to polyominoes, explore relationships among different polyominoes, and develop spatial reasoning by wrapping shapes around a cube and identifying which hexominoes form the net of a cube. While NCERT formally introduces nets of a cube in Class 7, this activity served as an exploratory learning experience for Class 4 students, offering early exposure through hands-on investigation using polyominoes.

The activity was conducted over three one-hour sessions, providing students with adequate time to explore, discuss, and reflect on their mathematical thinking.

Session 1: Introducing Parent–Child Relationships among Polyominoes

The first session began with a simple task: students were asked to join **two squares edge-to-edge** and determine in how many ways this could be done. Students initially joined the squares in multiple orientations—vertically, horizontally, diagonally, and in some cases, point-to-point. This provided an opportunity to reiterate the instruction that squares must be joined **edge-to-edge**.

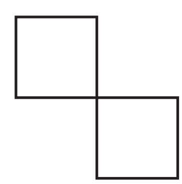

Figure 1: Students joined squares point-to-point as in this image

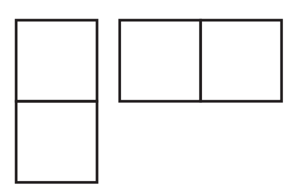

Figure 2: Students showed the given orientations

Keywords: polyominoes, nets, cube, tetrominoes, hexominoes

A follow-up question—*"Are these different shapes, or can one be rotated to form another?"*—prompted students to reflect on **rotation and uniqueness**. Through discussion, students concluded that despite different orientations, there was only **one unique arrangement**, as the other one was the rotation of it. Similarly, shapes that can be transformed into one another through reflection are regarded as the same. In other words, mirror images are not counted separately.

At this stage, formal terminology was introduced: **monomino, domino, tromino, tetromino, pentomino**, and so on, emphasizing that the number of unit squares determines the name. Students were then asked to draw **trominoes**. Although this was assigned as an individual task, students naturally engaged in peer discussions—comparing shapes, identifying rotations, and recognizing repetitions.

Through these discussions, students discovered that there are only **two distinct trominoes**.

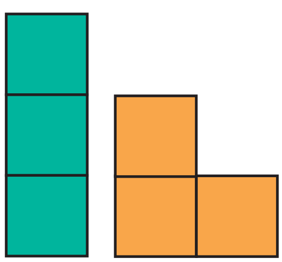

Figure 3: Students came up with these two trominoes

Next, students were asked to construct **tetrominoes**. Working collaboratively, they realized that there are exactly **five unique tetrominoes**, accounting for rotations and reflections.

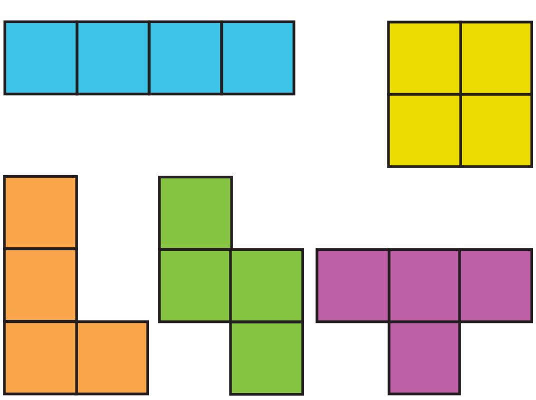

Figure 4: Students constructed the given tetrominoes

In an attempt to construct trominoes, one of the students came up with the following response, where the student tried to add a square to the domino, to form a tromino.

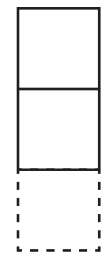

Figure 5: Example of a response a student came up with to construct a tromino using a domino

When students suggested adding a square to obtain a new shape, the idea of building larger polyominoes from smaller ones was introduced. In this process, adjoining one or more squares to an existing polyomino results in a new polyomino. In this context, a *parent polyomino* refers to an existing polyomino from which a new one is generated by adjoining one or more squares. The resulting larger polyomino is referred to as the *child*. This led to several insightful student observations; two such realizations were:

- A single parent polyomino can have multiple child polyominoes
- A child polyomino can have more than one parent

They represented these relationships visually using arrows on the board, as shown in *Figure 6*.

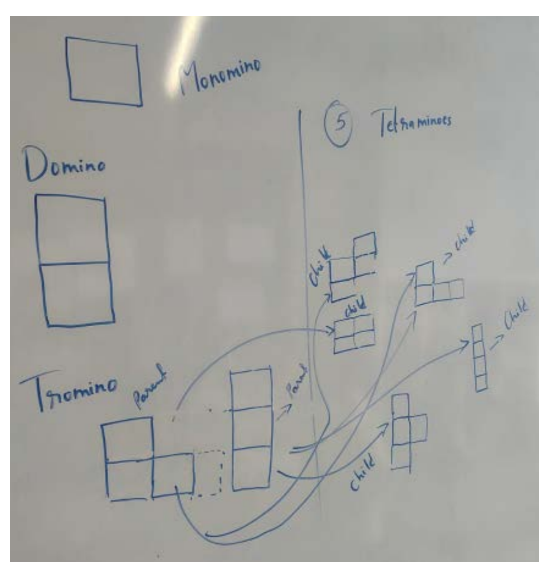

Figure 6: Introduction of the terms on the board and students suggesting parent-child relation using arrows.

Let us examine the relationships they observed:

Students noticed that the **L-shaped tromino** could act as a parent to **four different tetrominoes**, depending on where the additional square was placed (Fig. 1.7).

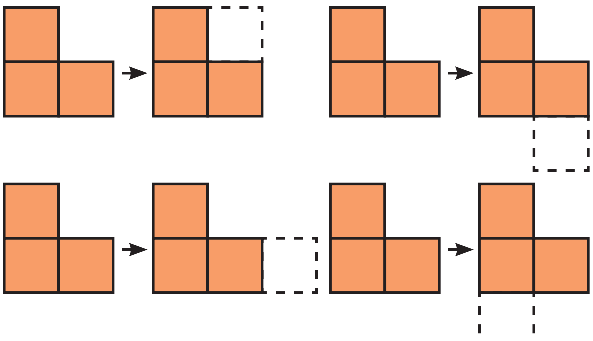

Figure 7: Construction of 4 different tetrominoes using an L-shaped tromino

This helped students realize that a single polyomino can produce multiple child polyominoes.

Similarly, when considering the **I-shaped tromino** as the parent, students found that it could generate **three different tetrominoes**, as shown in Figure 8.

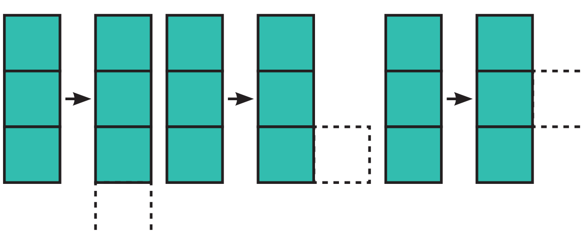

Fig 8: Construction of 3 different tetrominoes using an I-shaped tromino

From Figures 7 and 8, students observed that two tetrominoes — the **L-shaped** and **T-shaped** — can be formed from *both* trominoes. This helped them realize that a single child polyomino can have multiple parent polyominoes.

The discussion concluded with a student insightfully remarking:

> *" The monomino is the parent of all polyominoes".*

Exploring Pentominoes: Multiple Approaches

When asked to draw pentominoes, students adopted different strategies. Some rearranged five squares repeatedly to explore various combinations, while others extended tetrominoes by treating them as parents and generating child pentominoes. Several students illustrated parent–child relationships after drawing the pentominoes, using arrows and rough work (Figures 9 to 11).

These varied approaches highlighted students' growing comfort with systematic reasoning, spatial visualization, and recognition of rotational equivalence.

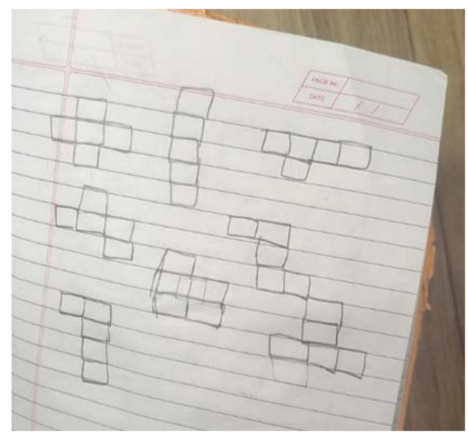

Figure 9: The student has drawn pentominoes without the help of parent-child relation

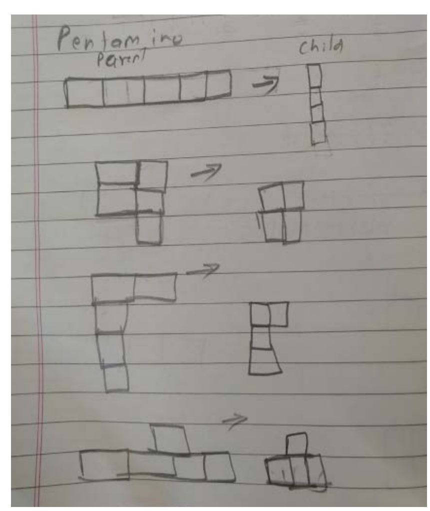

Figure 10: In this image, the names of parent and child are exchanged (could it be because the child thought that the parent had to be bigger than the child)

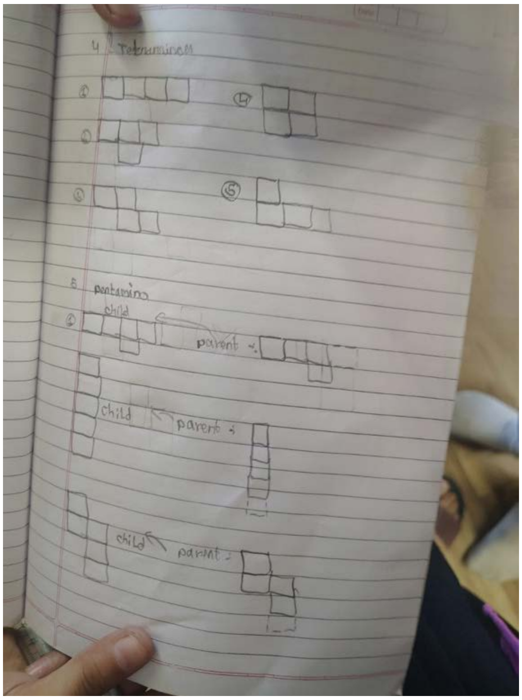

Figure 11: In this image, dotted line is used as rough work for drawing pentominoes

When some students felt they had exhausted all possibilities (and a few produced repeated shapes), I asked them how they could be certain that no additional pentominoes existed. This prompted initial attempts at justification. Students began informally examining the positions of the five squares and comparing shapes to see whether a new drawing was genuinely different or simply a rotated or reflected version of an existing one. Although this reasoning was not fully systematised at the time, the discussion introduced the idea of checking for duplication under symmetry and encouraged them to think beyond trial-and-error generation.

Session 2: Wrapping tetrominoes around a Cube

The second session focused on wrapping polyominoes around a cube. In this article, "wrapping" is defined as **covering as many faces as possible**, even if some faces remained uncovered; however, one face cannot be covered more than once, that is, overlapping is not allowed. Initially, many students interpreted "wrapping" as **completely covering all six faces of a cube exactly once**. This led them to reason using **factors and multiples,** concluding that:

- A tromino could be used twice to cover six faces
- A tetromino could not be used, as it would leave faces uncovered

This misinterpretation, however, proved productive. It revealed how students shifted their reasoning from orientation to numerical properties. Then, the meaning of the word "wrapping" in this context was clarified. The reason for this was to use the parent-child connection to generate hexominoes that were possible nets for the cube.

Students were asked to draw tetrominoes on grid paper, the teacher then cut them out, and attempted to wrap them around a cube. With physical cut-outs in hand, students began reasoning more effectively about **orientation and spatial fit**. Through exploration, they discovered that **four out of the five tetrominoes** could be wrapped around a cube, while one could not (Figure 12).

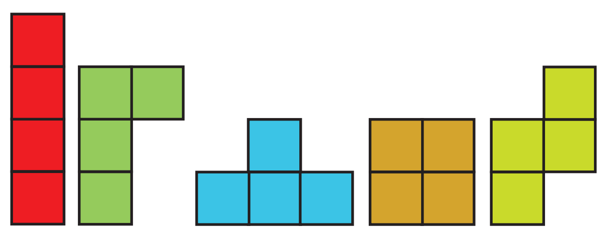

Figure 12: Students realized that the orange tetromino is the only tetromino which couldn't be wrapped around a cube.

Although making cut-outs enhanced understanding, it was time-consuming since scissors could not be distributed amongst students.

Session 3: Hexominoes and Nets of a Cube

To manage time more efficiently in the third session, students drew hexominoes in their notebooks while pre-prepared cut-outs were provided. Drawing all possible hexominoes proved challenging, but students were paired to encourage discussion. Partners debated whether shapes were rotations of one another and identified repeated forms.

Students were given approximately **35 minutes** for this task, and interestingly, some were able to construct **all 35 unique hexominoes** (Figure 13).

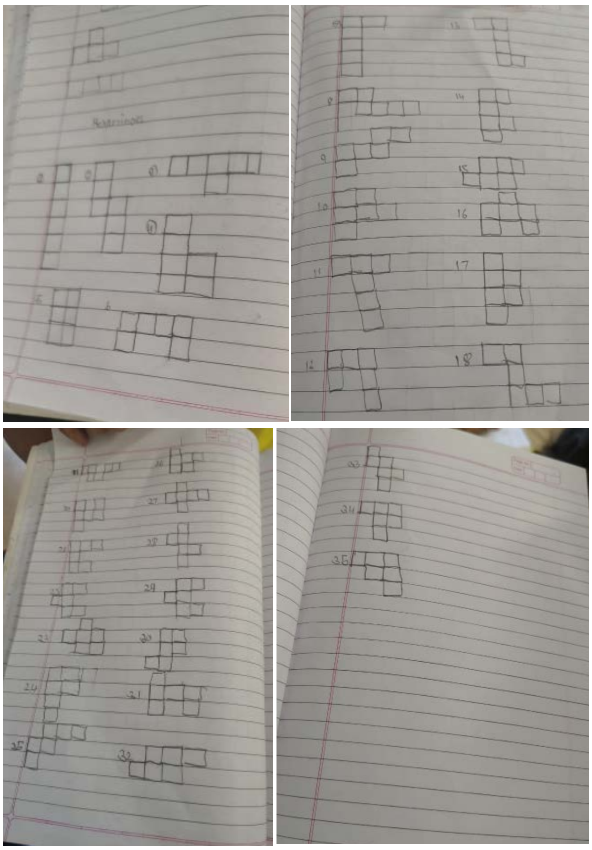

Figure 13: The given four images show all the possible hexominoes

Using hexomino cut-outs and cubes, students tested which shapes could wrap around a cube, setting aside those that could not. Out of the **35 possible hexominoes**, they discovered that only 11 could form a cube. At this point, the formal idea of a cube net was introduced.

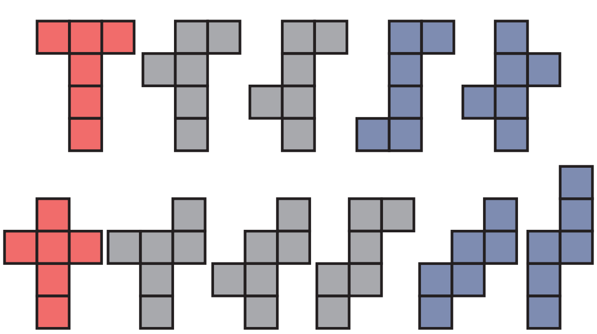

Figure 14: Hexominoes that can form nets of a cube

Students then folded their nets, experiencing the transition from **2D shapes to 3D structures** firsthand. Watching flat shapes transform into cubes marked a moment of excitement and accomplishment. The completed cubes became not just mathematical objects, but tangible celebrations of their exploration and learning — a meaningful takeaway from the activity.

Across the session, it was common to see pairs of students often asking questions such as, "What if we add the square in another position?" and discussing whether two shapes were genuinely different or merely rotations or reflections of one another.

During the activities on wrapping and tetrominoes, I provided linking cubes to help students test their constructions. However, this occasionally led to confusion, as the size of the linking cubes did not always match the dimensions of the paper cut-outs. I encouraged students to focus less on the exact size and instead consider whether the shape could conceptually wrap or cover the surface if scaled appropriately.

While attempting to wrap nets, when a square was left uncovered, students experimented by rotating the cut-out, repositioning the leftover flap first, and then attempting to fold it again. These trial-and-adjust strategies reflected their developing spatial reasoning and persistence.

Reflections and Learning Outcomes

Overall, the activity was highly engaging and intellectually enriching. Students demonstrated:

- A clear understanding of rotation and uniqueness
- Improved spatial reasoning while wrapping polyominoes
- Meaningful connections between 2D shapes and 3D objects
- Collaborative reasoning and mathematical communication

Suggestions for Future Implementation

Based on classroom observations, the following improvements are suggested:

- Prepare polyomino cut-outs in advance to save time and maintain engagement
- Ensure sufficient cube nets so all students can actively participate
- Pose more probing questions, such as:
  - → *Why do you say one shape is a rotation of another?*
  - → *How can we be sure two hexominoes are genuinely different?*
  - → *Have you encountered other situations in which the child is bigger than the parent?*

Such questions could deepen students' conceptual understanding and mathematical justification skills.

Conclusion

This three-session exploration using polyominoes offered Class 4 students a powerful and intuitive pathway to understanding **nets of a cube** well before formal curriculum introduction. By progressing from simple polyomino constructions to wrapping hexominoes around cubes, students developed strong spatial reasoning, an understanding of rotation and equivalence, and the ability to systematically explore mathematical possibilities. The use of parent–child relationships further deepened their structural thinking and pattern recognition.

Most importantly, discovering through hands-on investigation that only **11 out of 35 hexominoes** form cube nets transformed an abstract idea into concrete understanding. Folding successful nets into cubes provided a celebratory closure to the learning journey, reinforcing the connection between 2D shapes and 3D objects while fostering confidence, curiosity, and meaningful mathematical engagement.

Reference and citations

- Figure 3: Images of trominoes. Source: Wikipedia contributors, 2025, Tromino.
- Figure 4: Images of tetrominoes. Source: Alchetron, 2024, Tetromino.
- Figure 6: Students' observation on parent-child relation. Source: Author's classroom, 2026.
- Figure 7: Construction of tetrominoes using L-shaped tromino. Source: Adapted from Wikipedia contributors, 2025, Tromino.
- Figure 8: Construction of tetrominoes using an I-shaped tromino. Source: Adapted from Wikipedia contributors, 2025, Tromino.
- Figure 9: Drawing of pentominoes without using parent-child relation. Source: Author's classroom, 2026.
- Figures 10-11. Drawing of pentominoes using parent-child relation. Source: Author's classroom, 2026.
- Figure 12: Images of tetrominoes. Source: Puzzle Genius, n.d., Tetromino.
- Figure 13: Images of hexominoes. Source: Author's classroom, 2026.
- Figure 14: Images of hexominoes that are nets of cube. Source: Adapted from Wikipedia contributors, 2025, Hexomino.

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Review: The nRich Website

Reviewed by Sneha Titus

At Right Angles has already carried a review of the nRich website in their December 2012 issue (https://bit.ly/45X6F5Y). Why then a second review? I had two reasons for attempting this task again.

In March 2024, Azim Premji University Publications decided to bring At Right Angles back to its original purpose and intent – to be a quality learning resource for teachers/teacher educators of the primary and upper primary grades of the public education system in India. This review will therefore focus on resources for primary school. Secondly, the nRich website has been redesigned since the last review. Let's look at what has changed and what remains the same.

The home page https://nrich.maths.org/ is simple, uncluttered, inviting you to explore their free mathematics resources for students aged 3-18 years. Who is the 'you'? Teachers, students, and parents – and the tabs on top echo this with additional tabs for Problem-Solving Schools, Events and About nRich. It is interesting that this last tab doesn't feature the bio-details of those behind the website. Rather than focus on the fact that they are based in the Faculty of Mathematics at the University of Cambridge, it talks about their beliefs, what they think and why they do it and this diagram from the site (see Figure 1) says it all.

I discovered more about these terms as I explored the Teachers pages in some detail. My first hop was to the Early Years section and here I found tabs for Activities, Articles, Children's Thinking and Professional Development. Of course, I made my way straight to the last tab and signed up for their newsletter (see the tab at the bottom left of the page) which they promised would inform me about upcoming free webinars.

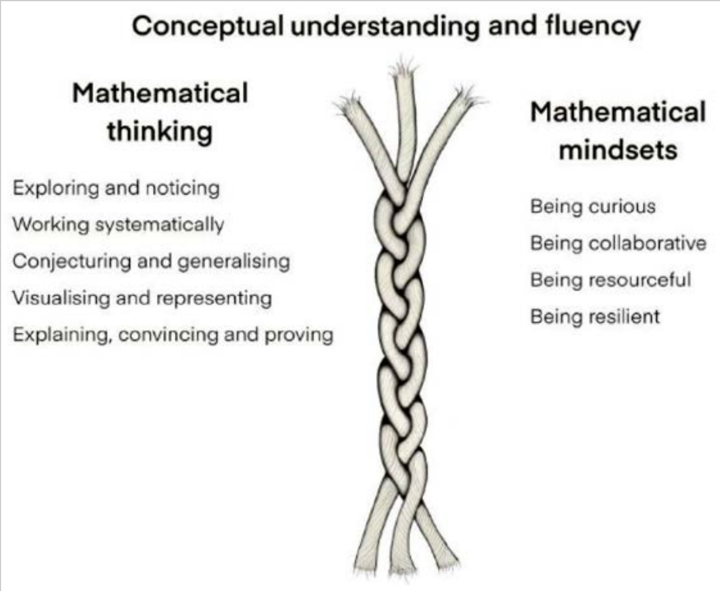


Figure 1: Source: https://bit.ly/46zI9Is

Next, I made my way to the Activities section and again, was impressed by the explanation of their Early-Stage Foundation Activities (EYFS) format. The activities here are built around what children (3-5 years) often do and how the adult could follow up on this. Let's look at an example from The Mud Kitchen https://bit.ly/4cmUV0E

Action: Designate a play-area where a small group of children can freely explore a mud-kitchen	
Stimulus	Children often enjoy digging in soil (mud), filling containers and emptying them, engaging in imaginative play and talking freely.
Reaction	Adults could provide an assortment of all kinds of objects from a kitchen - pots, pans, a water supply, large and small kitchen utensils, etc.
Cues- Describing	What's over here?
Reasoning	Why do you think this utensil works for this? Can you see anything that can be used there? Why would it be useful?
Opening Out	What would you like to do with this? Is that big enough for this?
Recording	Would it help to remember this if we took a photo?
The Mathematical Journey (Concepts and Vocabulary)	Counting Same and different Measure
Development and Variation	Sand pits, Water play
Resources	An outdoor area with soil, Kitchen utensils (pots, pans, spoons, spatulas, etc.), large drum (to use as a table), play cooker, small blackboard, water, plastic tubs, etc.

Recognising that the three strands of Conceptual Understanding and Fluency, Mathematical Reasoning and Mathematical Mindsets from Figure 1 are woven into this simple everyday activity was an 'aha' moment for me! Big words can be tailored to suit small folk 😊

The EYFS format helps to draw these strands out with the stimulus provided by what 'children often' (do) and the suggestions provided for the adult reaction. Mathematical Thinking is developed by using cues from the activity for Describing, Reasoning, Opening Out and Recording. Check out the descriptor for The Mathematical Journey which is best described in their own words.

The Development and Variation tab suggests similar activities, songs and poems, and the Resources section describes what exactly is needed for the activity. The activities are arranged by the domains of Number, Measures, Shape and Space, and Pattern. A word of caution- these songs and poems may not be culturally relevant to the Indian context.

What to do, how to do it, why we do it, what it develops and how to develop it further… what more does a teacher need for guidance? If we look at the public school system in India, there will certainly be issues with language. The website does not have translations for its English-language content and resources which are designed for UK/US curricula. School teachers in India may have to use Google Translate to avail of the resources available here. There are plenty of overlaps with the content in the Indian curriculum and teachers will themselves enjoy playing with the activities.

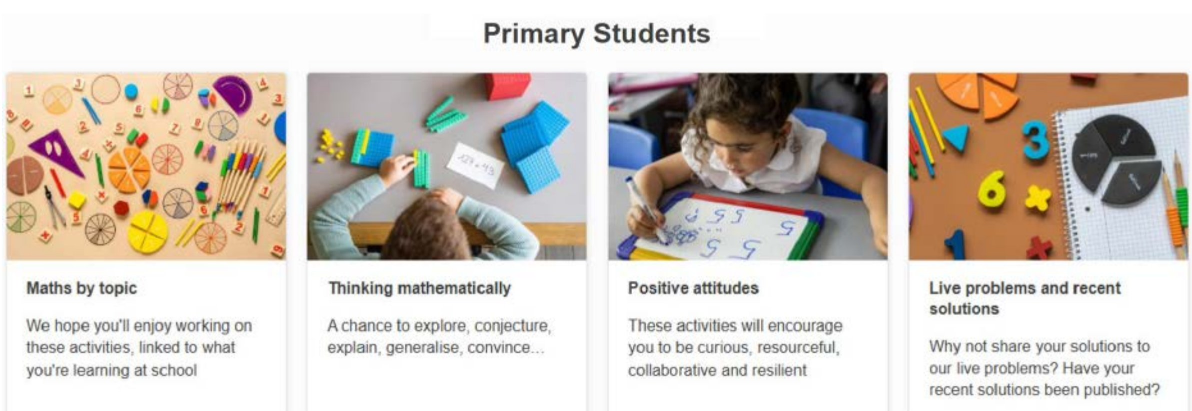
Again, classrooms with limited access to the Internet and to computers for children to use the interactive resources will be underserved by the website. On the plus side, most activities have a downloadable pdf with printable resources which may be used as handouts.

Clicking on the resources under the Teachers tab takes you to articles and activities arranged according to age for Early Years, Primary, Secondary and Post-16. Each activity is also arranged according to Challenge Level- of course this will vary depending on the student's ability but it is a rough guide for the teacher. Articles on classroom pedagogy, famous mathematicians and mathematical topics help teachers tailor their teaching for their class profile.

While researching for this review, I came across Colin Foster Mathematical Etudes https://bit.ly/4l34XWM where he suggests 'lovely, rich' alternatives to tedious tasks. His suggestions are open-ended though designed around a specific procedure. Drill and practice accomplished in subtle, enjoyable ways – what more can a teacher (or parent or student) want?

I have been personally influenced by Alan Wigley's article ' Models for teaching mathematics' https://bit.ly/47gTJIA where he cautions teachers about the 'path-smoothing model' and offers suggestions for the 'challenging model'. Issues which are common problems for Indian teachers are mentioned here -paucity of time and vastness of content to cover with students of differing abilities. However, in most Indian classrooms, transactions are teacher-driven, students are very used to following instructions. The suggestions given here may not apply to such classrooms. This brings me to the larger issue of teacher mediation. The activities here are intended for exploration, reasoning and problem-solving. I am not sure if in the Indian classroom, teachers are used to holding back from simplifying, explaining and giving away the solution. If they can be induced to wait, these problems and activities can drive more independent learning and perhaps even get students to enjoy instead of fearing mathematics.

I simply loved the Resources which are provided at the bottom of each section of the Teachers Page; there are printable, interactive and live resources- the last is a set of problems for which students are invited to send in their answers as seen in the last column of this screenshot of the link for Students (there are links for Primary, Secondary and Post-16).



There are many, many interesting activities and I have provided some links at the end of the page, but I want to end this review with seven reasons why I would strongly recommend this website.

1. It provides direction: Where do I want to go? What is my 'horizon' view for the teaching of mathematics?
2. It provides the map: How do I get there? What are the pitfalls and challenges? How can I find ways around these? How can I learn from the experience of others? Look at https://bit.ly/4r0R2lL It links topics taught at the primary level with activities and articles available on the website and illustrates my point without further explanation.

3. It provides resources: I found that while these resources may have to be tailored for the Indian context, they can be done so easily and are often low-cost. Also, the interactive activities are fun and easily accessible on the mobile phone. A major drawback is that instructions are in English; however, once these are understood, either by using a web-translator or with the help of an adult, students can stay transfixed for hours, playing at increasing levels of proficiency.
4. It demystifies jargon: Terms such as ‘mathematical mindsets’, ‘mathematical reasoning’ and ‘conceptual understanding’ are worthy goals but how can a teacher build these strands into her daily lesson plan? How can a parent engage in meaningful, yet fun play at home?
5. It delivers continuous professional development while helping teachers who reach out for that quick idea for the lesson. In addition, articles are in simple language and relate to the problems faced by teachers across the world, while providing interesting solutions and feedback from other teachers who have tried these solutions.
6. It encourages students to try problems, document and share their findings and it celebrates good thinking and problem-solving abilities.
7. It provides both horizontal as well as vertical content knowledge, enabling the teacher to go wide at a particular age-band while at the same time, connect to what is done in the previous and subsequent age-bands. For example, https://bit.ly/4rGkChj provides a teaching trajectory that helps teachers understand why understanding pattern is such an important pillar of the pedagogy of mathematics and how it can be developed from an early age.

Here are some activities that I would recommend:

Link	What is it?	Why I like it
https://bit.ly/4shndyb	It links a simple activity such as tidying up with mathematics	It is in the EYFS format explained above, so the pedagogical aspect is unmissable. And it is linked to good habits that teachers and parents try to inculcate in students.
https://bit.ly/4l46WKJ	A list of statements to be classified into Always/Sometimes/ Never True	Conjecturing and generalizing at the primary level becomes possible with such activities
https://bit.ly/3MPmCov	Data Handling exercise with post-its on the floor	It’s doable, it’s interesting and the visuals that emerge lead students onto pictograms and bar charts. In some Indian classrooms this has been tried with stick-on bindis
https://bit.ly/4cSxegL	Number skills with kinesthetic activity	Students explore, notice and learn about strategy
https://bit.ly/4u4DKat	Comparing 2 two-digit numbers	Not as simple as you think, the scoring depends on getting numbers which are as big as possible.

I hope you enjoy visiting and using these resources as much as I have!

SNEHA TITUS is Chief Editor of At Right Angles, which she has been associated with since its inception. She enjoys engaging in both language and mathematics. She has contributed to the Class 4 and 5 Sikkim Mathematics textbooks and to the designing of assessment for several age groups.

Sneha holds a master’s degree in mathematics and taught higher secondary school mathematics for several years and has authored chapters for NCERT Textbooks. Her current interests include learning more about computational thinking as well as exploring connections between craft and mathematics.

Call for Articles!

At Right Angles is a quality resource dedicated to mathematics education in India's public education system. It is specifically designed for teachers and teacher educators at the foundational, preparatory, and middle school levels.

We invite articles from mathematics teachers, educators, practitioners, parents, and students. If you are looking for a platform to contribute articles that support and enhance the learning experience of mathematics particularly for students approximately in the age group 6-14 years , we welcome your submissions.

Suggested Topics and Themes

Submitted articles should focus on curricular content applicable to Classes 1-8 and could:

- Explain and illustrate themes and topics outlined in the National Curriculum Framework for School Education 2023 (NCF-SE 2023).
- Specifically address challenges discussed in the NCF-SE 2023.
- Be substantiated accounts of the history of mathematics or the history of mathematical thinking.
- Include innovative worksheets or methods to engage students in drill and practice.
- Describe real-life applications of mathematics relevant to the child's context.
- Describe interdisciplinary activities or projects.
- Review puzzles or games with a practical connection to the syllabus.
- Offer guidance on selecting relevant content, including online resources.
- Develop pedagogical strategies for foundational numeracy as well as computational thinking.
- Assist teachers in implementing differentiated teaching practices.
- Review of Teaching Learning Material (TLM) or describe how to use local context, and local TLM in the math class.
- Provide material to help students bridge gaps in conceptual understanding.
- Address issues in assessment.
- Suggest ways to identify and address misconceptions in mathematics learning.
- Offer a list of problems along with discussions on their solutions and problem-solving strategies that are not commonly found in textbooks.

In addition to full-length articles, we also welcome shorter pieces that can include a variety of engaging content. These could be reviews of books, mathematics software, or YouTube clips that explore mathematical themes. Other contributions can be 'proofs without words', mathematical paradoxes, 'false proofs', or creative expressions such as poetry, cartoons, or photographs with a mathematical theme. We also welcome anecdotes about a mathematician or interesting examples of 'maths in craft, movies, etc'.

Articles may be sent to
atrightangles.editor@apu.edu.in

Please refer to specific editorial policies and guidelines on the inside back cover.

Policy for Accepting Articles

At Right Angles is an in-depth, magazine on matters of consequence to early mathematics and mathematics education. Hence articles must attempt to move beyond common myths, perceptions, and fallacies about mathematics.

The magazine has zero tolerance for plagiarism. By submitting an article for publishing, the author is assumed to declare it to be original and not under any legal restriction for publication (e.g. previous copyright ownership). Wherever appropriate, relevant references and sources will be indicated in the article.

At Right Angles brings out translations of the magazine in other Indian languages. Hence, Azim Premji University holds the right to translate and disseminate all articles published in the magazine.

If the submitted article has already been published elsewhere, the author is requested to seek permission from the previous publisher for re-publication in the magazine and mention the same in the form of an 'Author's Note' at the end of the article. It is also expected that the author forwards a copy of the permission letter, for our records. Similarly, if the author is sending his/her article to be re-published, (s) he is expected to ensure that due credit is then given to **At Right Angles**.

While **At Right Angles** welcomes a wide variety of articles, submissions that are found relevant but not suitable for publication in the magazine may be used in other avenues of publication within the University network, with the author's permission.

Specific Guidelines for Authors

Prospective authors are asked to observe the following guidelines.

1. **Engaging Introduction:** Write in a readable and inviting style, aiming to capture the reader's attention from the start. The first paragraph of the article should convey clearly what the article is about. For example, the opening paragraph could be a surprising conclusion, a challenge, a figure with an interesting question, or a relevant anecdote. Importantly, it should carry an invitation to continue reading.

2. **Catchy Title:** Title the article with an appropriate and catchy phrase that captures the spirit and substance of the article.

3. **Style:** Avoid a 'theorem-proof' format. Instead, integrate proofs into the article in an informal way.

4. **Balance:** Refrain from displaying long calculations. Strike a balance between providing too many details and making sudden jumps that depend on hidden calculations.

5. **Accessible language:** Avoid specialized jargon and notation that will be familiar only to specialists. If technical terms are needed, please define them.

6. **Use visuals:** Where possible, provide a diagram or a photograph that captures the essence of a mathematical idea. Never omit a diagram if it can help clarify a concept.

7. **Concise References:** Provide a compact list of references, with short recommendations.

8. **Exercises and Questions:** Make available a few exercises, and some questions to ponder either in the beginning or at the end of the article.

9. **Citation format:** Cite sources and references in their order of occurrence, at the end of the article. Avoid footnotes. If footnotes are needed, number and place them separately.

10. **Abbreviations and Acronyms:** Explain all abbreviations and acronyms the first time they occur in an article. Make a glossary of all such terms and place it at the end of the article.

11. **Labelling visual elements:** Label and number all diagrams, photos and figures included in the article. Attach them separately with the e-mail, with clear directions. (Please note: the minimum resolution for photos or scanned images should be 300 dpi).

12. **Precise references to visuals:** Refer to diagrams, photos, figures and tables by their numbers and avoid using references of these kinds: 'here', 'there', 'above', 'below', 'to the left', 'to the right'.

13. **Author Bio:** Include a high-resolution photograph (author photo) and a brief bio (not more than 50 words) that gives readers an idea of your experience and areas of expertise.

14. **British Spelling:** Adhere to British spellings – organise, not organize; colour not color, neighbour not neighbor, etc.

15. **Format for submission:** Submit articles in MS Word format or in LaTeX.

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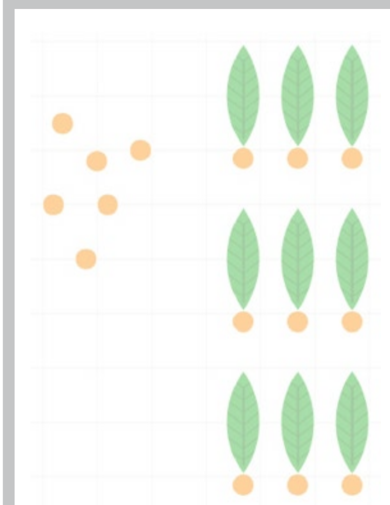
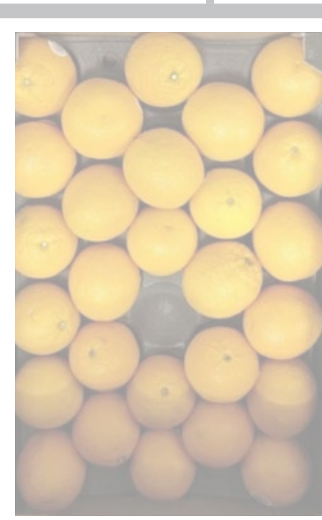


INTRODUCTION TO COUNTING



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INTRODUCTION TO COUNTING

PADMAPRIYA SHIRALI

A publication of Azim Premji University

INTRODUCTION TO COUNTING

Children's understanding of numbers begins naturally as they explore the world around them—even in infancy. Through repeated interactions with objects and the language used in their environment, they gradually build early number concepts.

In everyday situations, children encounter sets of objects and numbers like two and four, through interactions with familiar household items such as toys, plates, clothes, chairs, or food containers. Conversations around daily routines, for example, *"Did you eat two rotis?", "She has three dolls,"* or *"I bought six bananas and four apples"*—provide meaningful opportunities to experience numbers in context. These real-life experiences form the foundation of early numerical thinking.

Figure 1

Toddlers begin to associate number words with actual quantities, such as understanding that "two" refers to two apples or two toys. They also grasp comparative ideas, such as "more" and "less," for example, asking for more biscuits or fewer carrot slices. Initially, children view numbers as indicators of quantity. Gradually, they come to understand that numbers can also express position (second, third), serve as labels (House No. 104), or represent abstract measures (three years old).

Children possess an innate ability to visually perceive and compare small groups of objects—usually up to five—without counting, a skill known as **perceptual subitising**. Teachers can build upon this natural ability in order to support the development of effective counting skills. Additionally, it is worth noting that addition is inherent in counting, and subtraction is inherent in backward counting. Hence, these three concepts go together.

This pullout begins by highlighting the natural ability of both children and adults to instantly recognize small quantities (up to 6) without counting. It emphasizes strengthening this skill through various activities, alongside teaching effective counting methods for students in the three years of pre-primary and in Classes 1–2. Subitising plays a key role in early mathematics education, as it helps children to build mental images for numbers and visualize number facts. It develops an intuitive understanding of numbers, enhancing mental arithmetic skills, goes beyond rote counting towards flexible thinking about numbers, and helps to recognize patterns in numbers that support more advanced concepts in mathematics.

Keywords: number, pre-number skills, subitising, counting, numeracy, counting principles.

Not as simple as 1-2-3

Teachers at the foundational stage may observe that as children progress in their counting and number understanding, they often display certain common misconceptions and errors, such as:

- Misjudging amounts based on how objects are spaced or arranged.
- Not understanding that the last number counted represents the total quantity (the **cardinality principle**).
- Struggling to recall the correct counting sequence.
- Confusing the order of the number-names when counting forward or backward (the **stable order principle**).
- Making errors in one-to-one correspondence, such as skipping or double-counting items.
- Failing to grasp the **conservation principle** that quantity remains the same despite changes in arrangement.
- Underutilizing visual perception when comparing small sets of objects. Children may rely less on visual perception when comparing small groups of objects. After learning to count, they often believe they must count every time, even when recognizing the quantity visually—such as identifying a group of four objects.

Our teaching approaches must be designed to prevent these errors and actively support children in overcoming them as they learn to count and understand numbers.

Table 1

ACTIVITY 1: DEVELOPING PERCEPTUAL SUBITISING SKILLS

Objective: To support and assess children's ability to perceptually subitise—recognising small quantities (typically up to 5 or 6) instantly, without counting.

Materials: Dot flash cards, standard dice

Procedure

Perceptual subitising is the immediate recognition of a few items. For instance, when a child sees four dots on a die, they can say "four" without counting each dot.

Use dice as a natural tool for reinforcing these patterns, especially for quantities 1 to 6.

Figure 2

Prepare flash cards displaying various spatial arrangements of 3, 4, 5, and 6 dots.

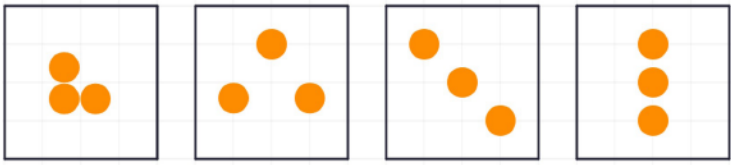


Figure 3: Example of flashcards for 3 dots

Use consistent dot sizes and spacing to ensure clarity. Briefly show each card (for 1–2 seconds) and ask children to say the number they see. This encourages visual pattern recognition rather than counting.

Extension

Once children confidently identify quantities from 1 to 6 across different configurations, introduce flash cards with 7 to 10 dots. An example for 9 is given (Figure 4).

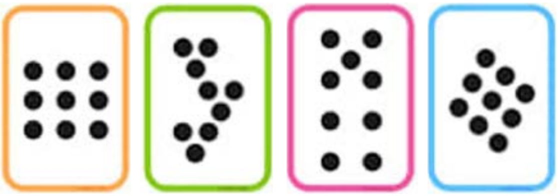


Figure 4

These cards may require more complex mental grouping and lay the foundation for conceptual subitising.

Conceptual subitising involves recognizing larger numbers by mentally grouping smaller sets within them. For example, seeing nine dots as three groups of three. This type helps children understand part-whole relationships important for addition and subtraction.

ACTIVITY 2

Objective: To support and evaluate children's development of perceptual and conceptual subitisation.

Materials: Hands and fingers, ten-frames, ten-frames with dots

Let them start with numbers below 5 initially. For example: Show 3 in different ways. (Figure 5)

Children naturally use their hands and fingers to show numbers up to ten. They begin to recognise number patterns formed by finger arrangements, such as seeing eight as five plus three or four plus four.

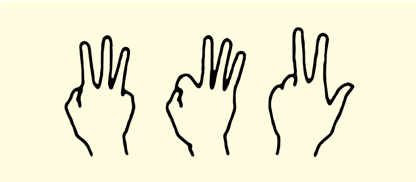


Figure 5

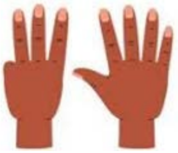


Figure 6

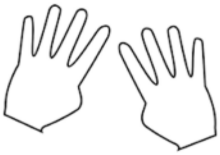


Figure 7

A ten-frame (Figure 8) is a rectangular grid made up of two rows and five columns, totalling ten sections or boxes. Children use it to place counters or markers to represent numbers up to ten, helping them develop number sense, understand one-to-one correspondence, subitising, and visual representations of numbers.

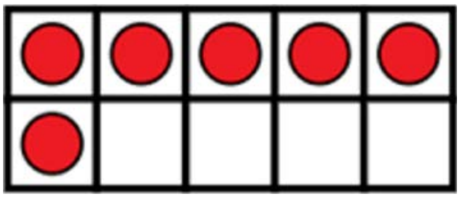


Figure 8

Let children place counters on these frames in various patterns and call out the number of counters. Its layout helps children identify numbers larger than five more easily.

This can be followed by using charts or flashcards with various number patterns displayed on ten-frames to enhance their ability to subitise both perceptually and conceptually.

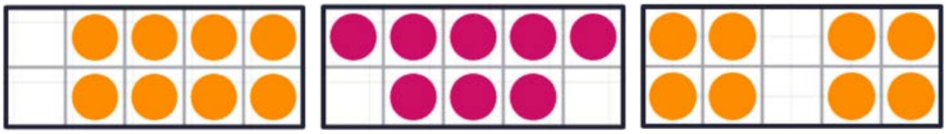


Figure 9: Flashcards showing the number 8 using a ten-frame arrangement

As children become more comfortable with the ten-frame, they may begin to view the cards in different ways. For instance, some might instantly recognize a pattern such as 4 and 4, while others may notice that two counters are missing from a full frame and conclude that the number is 8. It is essential to allow students the space and freedom to develop their own strategies for interpreting the frame. The teacher can encourage deeper thinking by asking questions like, "What makes you think it's 8?" or "What would happen if the bottom row were empty?"

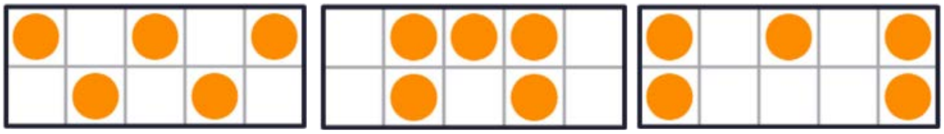


Figure 10: Flashcards showing the number 5 using a ten-frame arrangement

Extension: Showing Numbers in Different Ways

Hold up five fingers, display a giant dice, or show a large numeral, and then ask children to represent that number in as many ways as possible. Provide countable items and resources so they can experiment with different representations.

Possible materials:

- Seeds, counters, small toys, large blocks, multilink cubes (cubes that can be connected)
- Dot patterns such as on dice and dominoes
- Structured materials such as ganitmalas, 10-bead strips, Montessori number rods
- Everyday packing items such as egg boxes or crayon boxes, which have slots
- Number symbols, such as number lines or 100 square grids

Encouraging Mathematical Thinking

There are three 5s shown in Figure 11.

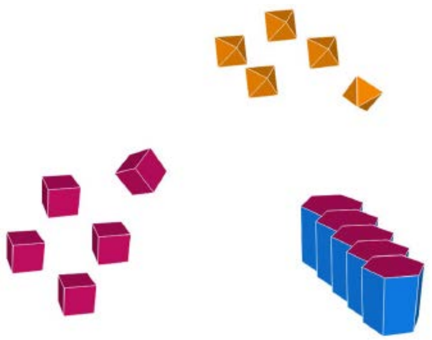


Figure 11

Describing

- How does this five look different from that five (point to another group)?
- What does this pattern of five (point to one group) look like?
- What can you see?

Reasoning

- How do you know these are all the same number?
- How does each five in Figure 11 look different from the others? What is the same, and what is different about these fives?

Opening Out

- How can you make five with two hands?
- Can you show me five using your fingers in another way?

Figure 12

Game 1: Hide-and-Reveal Counter Game

Children are naturally drawn to games of hiding and discovery.

Place one, two, and three counters beneath three separate but identical bowls. Briefly reveal the contents of one bowl, then cover it again, and ask the children to identify how many counters they saw. This encourages instant recognition.

ACTIVITY 3

Objective: To support and evaluate children's development of the conservation of number.

Materials: Counters of 2 different sizes or seeds of two different sizes, such as rajma and channa

In comparing two groups of objects, students may focus on the relative size of the objects or the way they are arranged rather than the number of items. They may see a collection of 4 objects arranged at a distance from each other as being more than a collection of the same 4 objects packed closely together.

Figure 13: Conservation Principle

The teacher can use different arrangements of counters (such as shown in Figure 14) and ask students to compare the two rows or two groups and identify pairs which show the same number, to ensure that the students have developed conservation of number.

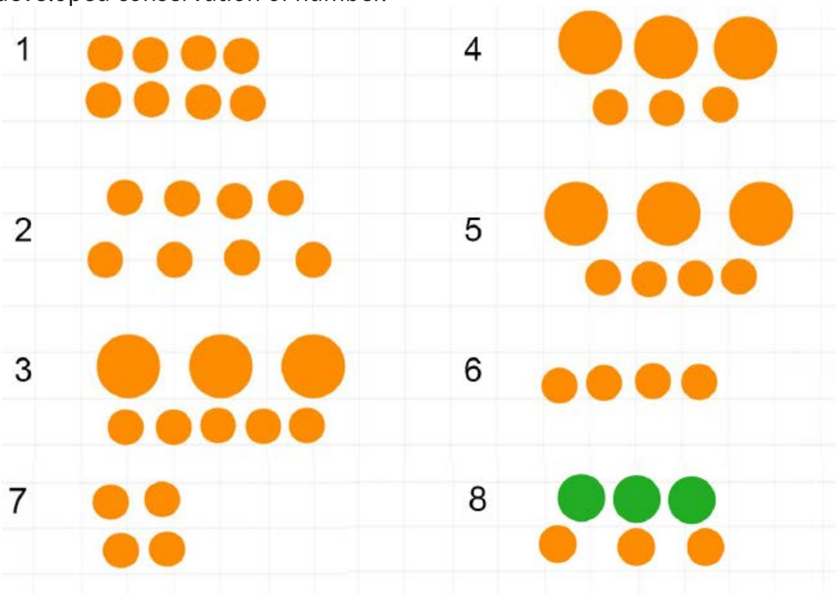


Figure 14: Exposure to such an activity will help students grasp the conservation principle that the quantity remains the same despite changes in arrangement.

In the next activity, we approach understanding the cardinal value of a set, which involves understanding that the value doesn't change unless something is added or taken away, an element of conservation of number.

ACTIVITY 4

Objective: To build a sound conceptual understanding of numbers through the notion of **'one more'**.
To understand the **cardinality** principle.

Materials: Set of objects (colourful beads/cubes), string of 10 beads, 10 interlocking cubes

The issues referred to in Table 1, (common errors while learning to count) can largely be resolved by explicitly introducing the concept of 'one more' at first and emphasizing it till the number relationships are fully grasped. This is followed by the concept of 'one less'. For example, 5 is 1 more than 4 or 9 is 1 less than 10. This approach slows the counting process, allowing time to point to each object (one-to-one correspondence) and highlight the counted group (cardinality principle).

The following conversation centred around Figure 16 will illustrate this approach.

Figure 15: A string of 10 large beads is a good aid to teaching counting in order.

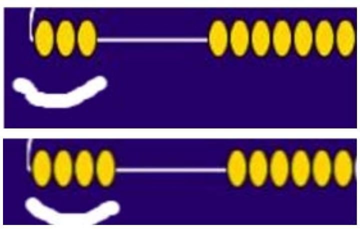


Figure 16

'Let's start. This is 1.' (Point to the first bead.)

Now move one more bead across. 'How many beads are on this side now? That's right, 2!' (Point to the two beads.)

Slide another bead over. 'How many beads are here now? Yes, 3!' (Point to the three beads together.)

Then slide another bead and point to the four beads together.

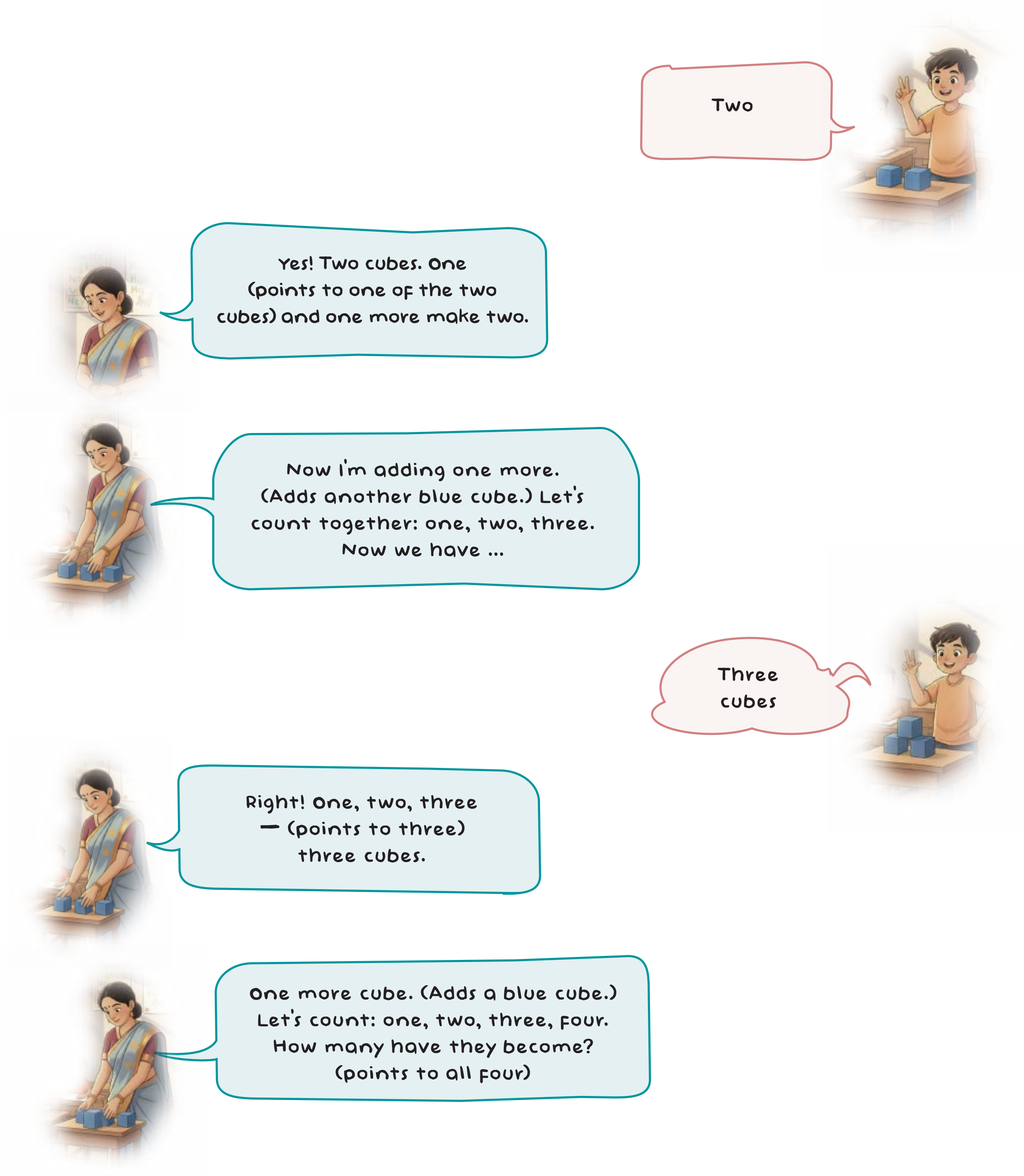
Repeat this activity by counting in different directions—right to left, left to right.

Later, the string can be held vertically and the counting can be from top to bottom, or bottom to top.

While working with cubes, a typical conversation between the teacher and the student would look like this:

Holding up one blue cube

Places it on the table.



Note: When teaching counting and number names, it is often more effective to divide the learning process into two stages. In the first stage, children are introduced to the numbers up to 5. This helps them grasp small quantities, understand basic counting, and become familiar with the corresponding number names. Once they are comfortable with this foundation, the second stage can focus on numbers from 1 to 10. Gradually extending the range allows children to build confidence, recognize number patterns, and strengthen both their counting and number recognition skills.

Game 2: Treasure Box

Materials: A cardboard box for holding small objects (such as shells, pebbles, or counters).

Instructions:

Display a few objects (for example, three shells) and count them aloud with the children: "1, 2, 3."

Place the counted shells into the opaque box while children watch.

Ask the children: 'How many shells are in the box?'

Then continue the activity by adding more shells to the box and prompt further thinking:

How many shells will there be if I add one more?
How many will there be if I add two more?
How many will be there if I remove one?

Encourage children to respond based on their counting and mental addition skills before checking by opening the box.

ACTIVITY 5

Objective: To build a sound conceptual understanding of zero through the notion of 'one less'. Counting backward, using language such as one less, zero, none.

Materials: Numeral cards (10 to 1)

Scenario: Engaging Children through Song, Story, and Number Play

This activity can be enacted by students while reciting any rhyme that counts from 10 to 0. Here is one possible rhyme.

"Zoom zoom zoom,
We're going to the moon!
Zoom zoom zoom

We'll be there very soon!
10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0. Blast off!"

Figure 17

This rhyme uses repetition and storytelling to make counting backwards accessible and enjoyable for young learners. Let children stand in a line behind one another, holding numeral cards 10 to 1, in decreasing order. Teach them the rhyme about going to the moon. Let them join in singing and acting out the story. Begin to sing the rhyme slowly. At the end of each verse, the child who is leading will leap and leave the column. Ask children to share aloud how many remain each time. *'We were 10. One less. Now 9', 'We were 9, one less. Now 8.'* Children will find the answer easy to predict, as each time it is one less.

Pose questions:

'What happens to the number of children each time one zooms away?'

'If two zoom away, how many are left? How do you know?'

'What if all zoom away at once? What if more children join?'

The teacher can evaluate the students by asking the children to model the activity with their fingers.

Evaluation

Pose riddles based on classroom context: You can see only one of me in the classroom! Who am I? You can see only two of me! Who am I? Let students give examples such as a blackboard or a door for the number one.

Home context: Pose questions that help them reflect on some objects at home. Discuss the number of members in the family. Ask if the number of cots is the same or more than the number of family members. Can they think of any objects that are more or less than the number of family members? Some objects in the house are usually only one, some may be two, and so on. Ask children to find objects in the house for which there is only one such object, only two, and so on.

Notes: Motivation

Children usually enjoy counting and often do it naturally in everyday situations. However, within the classroom, the teacher may sometimes need to create meaningful reasons to count. Encourage curiosity by asking questions such as, "How can we find out how many are here?"

Provide varied opportunities for counting:

- Count objects arranged in a line or scattered randomly.
- Count drumbeats, claps, or the number of jumps it takes to cross the room.
- Compare the number of letters in each child's name.
- Encourage estimation of small groups of objects and verify by counting.

Act out number-based stories like *Panch Pandavas* or *Snow-White and the Seven Dwarves*.

Missing magic number: Show a group of objects, secretly remove a few, and challenge children to find the "missing magic number."

ACTIVITY 6

Objective: To compare quantities using terms such as 'more' and 'less'.

Materials: Set of objects (colourful beads/cubes)

Collections: Children love to collect objects from nature. It could be stones, seeds, flowers from nature, or beads, cubes, and toys in the classroom. The teacher can give a container to each child or one container to be shared by 2 or 3 children. They can collect a few (between 5 and 10) objects of the same kind in their container.

Figure 18: A ganitmala helps in counting forward and back and in comparing numbers

Initiate a conversation: *'Tell me about what you have in this basket. How many? What happens when you put in another? What happens if you give me some of them?'*

'*Let's look at what is in these two containers. Do they have the same number of things? What makes that one different?*'

In a daily situation, talk about **'as many as'**, for example, '*The number of stools is as many as the number of students*.'

Objects in hand: Ask children to hold more objects in one hand and less in the other hand.

Switching sides: Draw 2 circles and ask two children to stand in one circle and five in the other circle. Pose the question 'How can we have more in this circle (pointing to the one with 2) and fewer in the other?' Do the students find more than one way of solving this?

Pose **open-ended** questions: '*Can you show more than that number? In how many ways can you show more?*'

'*Can you show less than that number? In how many ways can you show less?*'

What should we do to make these two containers hold the same number of objects? Children have an implicit idea of balance and will come up with various suggestions.

Figure 19a

Figure 19b

Note: Related relationships should be emphasised while comparing groups. '*There are more pencils than crayons. So, there are fewer crayons than pencils.*'

Usage of **one-to-one correspondence:** Children may be able to identify more or less for small sets through visual perception. However, as the numbers grow, the usage of one-to-one correspondence is the approach to be used.

Note: Before entering school, children usually learn to count a small number of objects (usually, up to 10). In school, this understanding of counting is further strengthened through structured teaching methods. Activities 4, 5, and 6 may be introduced before the subitisation activities 1, 2, and 3. The activities need not follow a fixed order, as some can progress in parallel.

ACTIVITY 7

Objective: To build effective counting techniques.

Materials: Set of objects (colourful beads/ cubes)

Children come to understand from their diverse experiences that the order in which they count objects does not affect the total count. This concept is known as the stable order principle. To grasp this, children need multiple opportunities to count the same set arranged in different ways, helping them realize that changing the counting order does not alter the total. Teachers can provide the experience of counting rows of objects such as the number of tiles on the floor, or shoes on the rack or books on shelves from multiple directions.

Although counting usually begins anew—from 1 up to the end of the set—effective counting often involves using prior knowledge or strategies such as partitioning, counting forward or backward, and grouping.

Examples of these strategies include:

- Partitioning and counting
- Matching and counting
- Grouping and counting
- Counting forward and backward

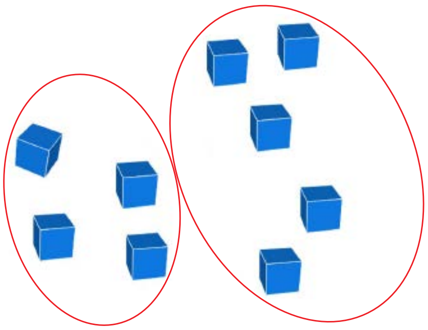


Figure 20

Partitioning and counting: A child may partition a group of objects into small manageable sets and use perceptual subitisation to arrive at the number.

Example: To count the collection of given cubes, child may mentally partition it into two sets and arrive at 9.

Matching and counting

How many berries?

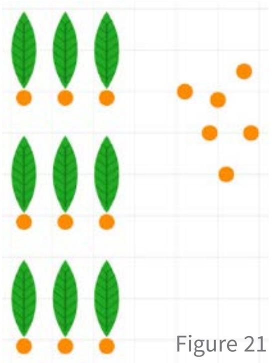


Figure 21

A child may recognize that there are 9 leaves and that there are 9 berries that are connected (matched) to each,and then count the remaining berries, arriving at 15 berries. (9 and 6 is 15.)

Grouping and counting

How many shapes in Figure 22? The child may decide to treat them as 3 groups with 3, 4, and 5 shapes in each group and count them together, recognising 3 and 4 together as 7 and then adding on 5 or recognising 5 and 4 as 9 and adding on 3.

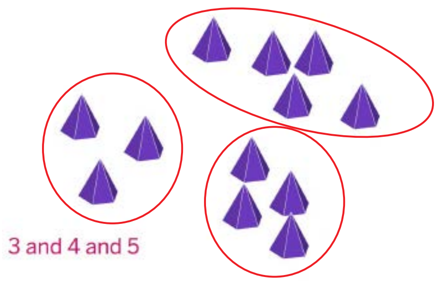


Figure 22

Counting forward and backward

How many circles in Figure 23?

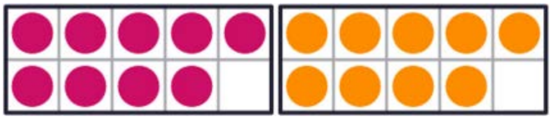


Figure 23

A child may recognise the configuration as 9 and 9 and count forwards from 9 onwards as 10, 11, 12, ... 18. Or may recognise the two frames as 10 and 10, and go 2 steps backwards as 20, 19, 18.

It is good to demonstrate these concepts through multiple materials. Ganitmala is to be used to reinforce the idea.

In forward counting, watch whether children are recognising the group and counting forward

Note: Avoid the use of the word 'plus/minus' and the symbols '+/-' in the early phase.

ACTIVITY 8

Objective: Counting using Patterns.

Create simple patterns such as shown in Figure 24 and ask them to count the shapes. Children may use their pattern recognition and classification skills.

How many pink circles? How many orange circles?

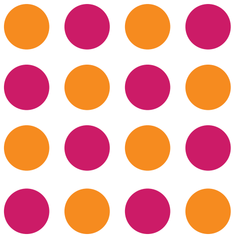


Figure 24

Figure 25

Construct patterns from right to left, left to right, top to bottom, etc.

Explore a variety of patterns, linear borders, and growing patterns.

Figure 26 shows a more complex pattern! How many blues? How many reds? How many yellows?

Ask the children to explain how they obtained their answers.

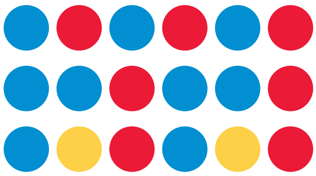


Figure 26

ACTIVITY 9

Objective: Counting exposure through everyday experiences.

Classroom: Counting the number of students present, counting materials like shapes, beads, etc., while tidying up the classroom at the end of the day.

The teacher could make number labels for seats.

Counting days remaining for a particular event on the calendar, or counting hours on the clock are ways to count using visual materials such as the calendar and the clock which are present in most classrooms.

The following rote counting ideas help students learn the number names in the correct sequence.

- Have students count forward and backward, one to nine.
- Let them count on from a given number
- Let them count between two given numbers

Common games: Snakes and Ladders, counting in hide and seek, and local versions of hopscotch (Kunte Bille/ Kith-Kith) with numbers from 10 to 20.

ACTIVITY 10

Objective: To encourage students to find ways of counting in new situations.

Materials: Pictures of objects numbering between 10 and 20.

Collect pictures of objects that may or may not be ordered in any manner. Pose the challenge of counting them. Observe the strategies that children are using.

Ask, *'How many are there? Show me how you counted them. How did you keep track of which ones had been counted?'*

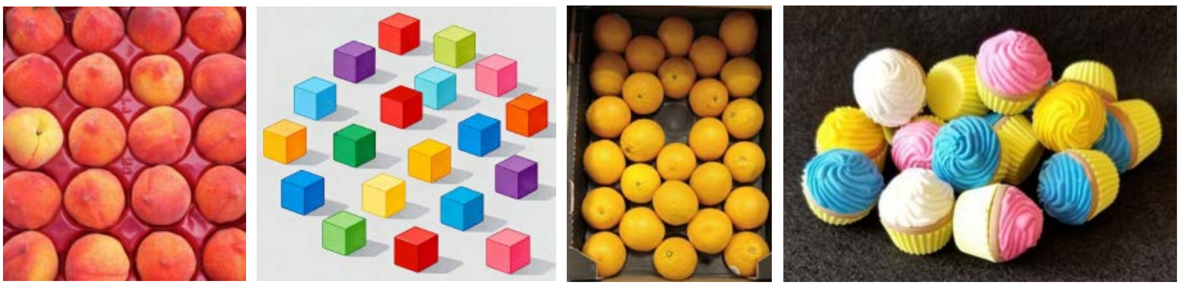


Figure 27

Figure 28 shows three groups of fruits. Which two groups together have 10 fruits?

Figure 28

Children enjoy counting, and teachers can create meaningful, fun activities and contexts in which children exercise their skill of counting.

Counting can take place in a variety of situations and with different kinds of objects. Some contexts involve static, movable objects—such as cubes or pencils—that children can physically handle during counting activities. Others might include moving objects, such as vehicles passing on a road. Counting can also refer to games, like the number of times a ball is bounced. Here are some examples:

- Children can be encouraged to count objects like the number of bicycles (cycles) passing by on the street.
- When counting objects that cannot be moved (for example, items shown in a worksheet), it becomes harder for children to keep track of which ones have been counted. It's important to use strategies such as marking or colouring.
- Counting actions (like jumps, claps, or bounces) is even more abstract, as these events do not leave a visible or tangible trace. Recording each occurrence—perhaps through tally marks—can help children keep track.

These activities will help the children to move from concrete object counting to abstract counting and later on to data handling.

The introduction to the concept of counting typically begins in early childhood education programs. In many Indian states, children become familiar with numbers up to 9 before they enter Class 1. During Class 1, they continue to develop their understanding of counting, numbers, and numerals—starting with numbers up to 20 and gradually progressing toward 99.

When teaching and learning numbers, teachers need not be strictly bound by the prescribed learning outcomes. If children show readiness, teachers can extend their learning beyond the stated outcomes.

This article addresses both the early childhood or foundational stages as well as Class 1. Teachers of Class 1 can use subitisation activities to revisit number concepts. Recognizing patterns in geometric arrangements of small numbers (such as 8 or 9) supports this process. Children develop this ability naturally through hands-on experiences—arranging pegs on pegboards, forming tile squares, or creating symmetric dot patterns. However, the emergence of this skill cannot be prescribed as a stage-specific learning outcome; it develops gradually and at different stages for different learners. Teachers may also choose to present these activities in the form of games to make learning more engaging.

Reference:

How Much or Till What: When and Why?

Review of Ten-Frames https://bit.ly/4r79Qkb

Note: AI has been used to generate some of the images in this article

Acknowledgement:

https://nrich.maths.org/articles/subitising

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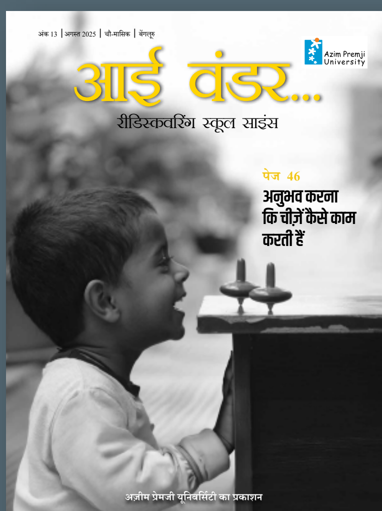

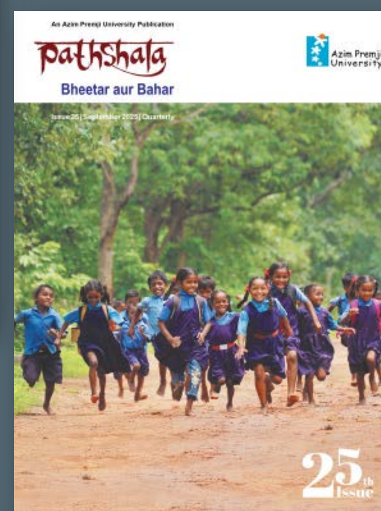

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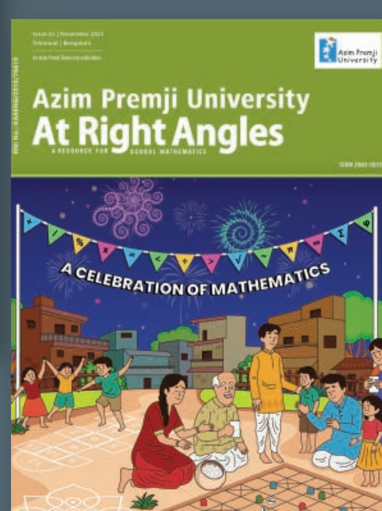

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