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Azim Premji University At Right Angles

A RESOURCE FOR SCHOOL MATHEMATICS

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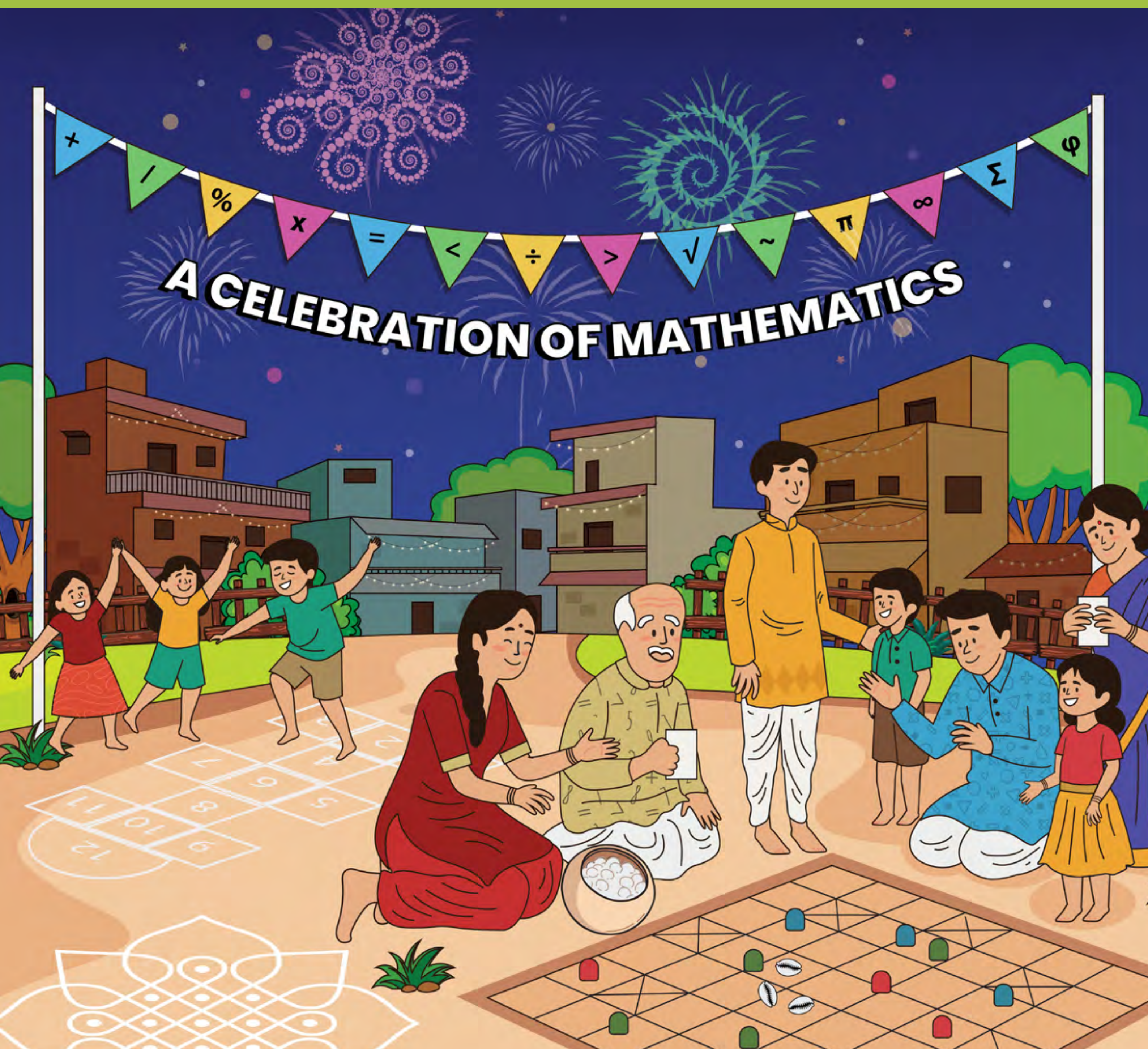

Festivals are a time when we celebrate all that is beautiful around us. Have you noticed how much mathematics there is in all that we do at such times?

Think of the patterns in our clothes and in the 'thoranams' that we hang up, the 'pookalams' and the 'rangolis' that grace our floors, the music that rings out at such times, even the food that we prepare and how we serve it.

We see numbers, shapes and sizes in geometry, patterns in numbers and in music, the use of data in our planning, including the catering for unknown variables!

Mathematics comes together in a harmonious whole in the celebrations of life and in the celebration of life.

From the Editor's Desk . . .

Dear Readers,

And just like that, 2025 has sped by and we are on the last issue of the year! **At Right Angles** has had a year of learning- not just of mathematics, but of the gaps and needs in the pedagogy of primary school mathematics. We examined the interweaving of craft, of art, of technology, of our environment and the occupations and preoccupations of our day-to-day lives with mathematics. And in the November 2025 issue, we focus on the celebration of mathematics! We are very conscious that there is a large population of students and adults who scoff at this idea- a famous quote goes, '*I have a love-hate relationship with mathematics- I hate to love it, but I love to hate it.*' A **celebration** seems completely contrary to such an attitude; so we wanted to examine how we could have a meaningful celebration that was inclusive, catered to different abilities and, most importantly, changed this mindset about mathematics.

Festivals promote diversity, they bring different stakeholders into dialogue, they increase creativity, they offer opportunities for 'owning the subject'. In short, they transform the classroom- at least for the day! How can we seize such opportunities to bring lasting change? Can we carry the festivities of the National Mathematics Day on December 22nd of every year into the academic year ahead? Then they are truly worth celebrating. The two Features articles and the Pullout of the November 2025 issue give you different aspects of, and ideas for, such celebrations.

This issue's Classroom section begins with Smruti Smarak Panda's thought-provoking article *What we Steal when we Teach*. How do we teach without detracting from learning? Jeenath Rahaman's story about young Maya's explorations with numbers and how she made *Little Childhood Discoveries about Divisibility* emphasizes the importance of student-led learning. And Karan Singh continues – *Teaching Area and Perimeter through Experience* describes a teacher training session in which teachers learnt how not to dictate formulas but to explore these concepts through contextual examples.

Kshama Chakravarthy's article *How do I Know They Got It?* builds in some good debugging of teaching practices. How can we detect and address the errors which creep into the process of concept building in a student's mind? Do take her up on her generous offer to build on this article with actual intervention initiatives in the classroom.

We end this section with Swati Sircar's report and follow up on Aakefa Basri's lesson on *Fact Families* – how can addition and subtraction facts build connections among numbers? Which gives us a great start to the Joy of Mathematics section- a *Cross Number Puzzle* after a very long time, this one based on Addition Fact Families. R Mohan describes interesting explorations with Square Tiles, and we conclude with a first for us- a mathematics play script and Padmapriya Shirali's reasons (based on her experience) why such plays draw even the most reluctant of mathematics students on to the centre stage of the mathematics classroom.

We thank the readers of At Right Angles for the different responses to Carelin Christopher's puzzle, Art in Numerals. Some of these are featured in this issue; both contributors sent several suggestions and we had to pick from these for want of space.

The Pullout and the cover of the **At Right Angles**, July 2025 issue focused on how weaving patterns could connect to school mathematics. We are delighted that the new NCERT Class 5 textbook also carries ideas for the same and we hope that teachers will enjoy weaving mathematics into the life of the class.

John von Neumann famously said, 'In mathematics, you don't understand things. You just get used to them'. We want to use that to urge you to make the celebration of mathematics with not just a one-day affair, but a way of life.

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At Right Angles is a publication of Azim Premji University which provides quality mathematics learning resources for school teachers. It intends to facilitate more experiential and meaningful teaching-learning processes, not just inside the classrooms but also in the broader context of school processes. To celebrate purposeful and passionate teaching, At Right Angles showcases practical insights grounded in the realities of India and its diverse communities.

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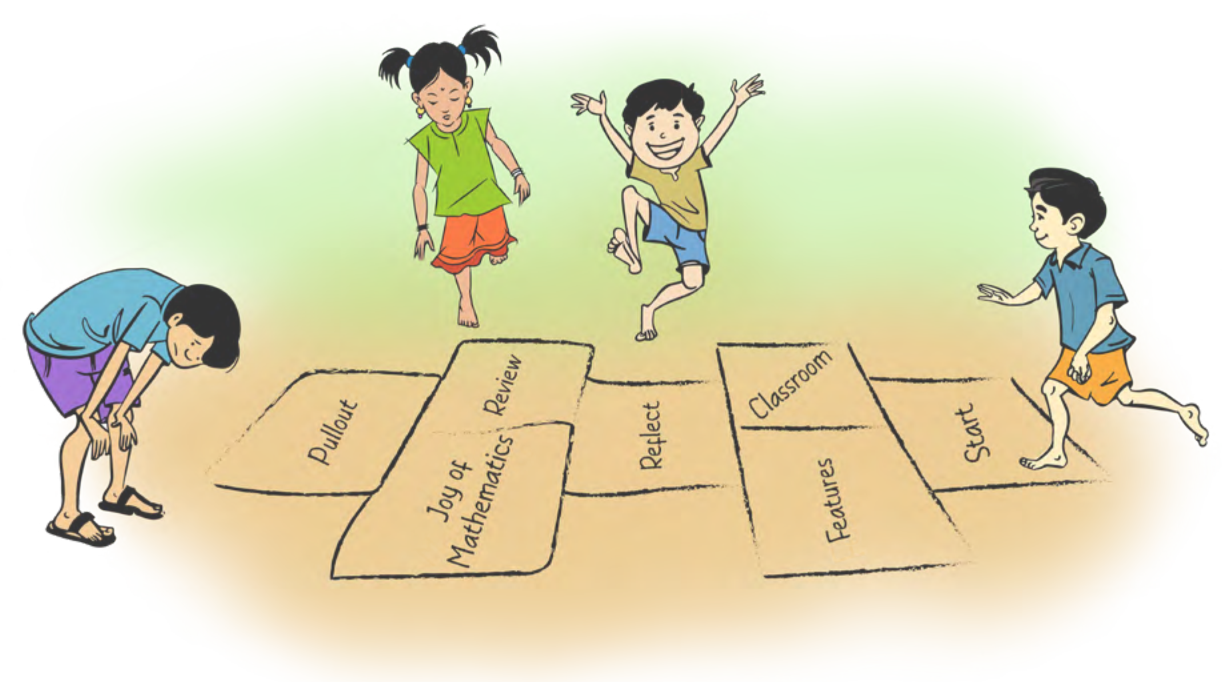

Mathematics Day Once More . . . . .

Compiled by the Editors, At Right Angles

As December approaches, institutions across the country get ready to celebrate the National Day of Mathematics on or around December 22, the birth date of Srinivasa Ramanujan. The At Right Angles team thought it was a good idea to ask those in the field to share their opinions and suggestions on the celebration of such a day.

These were the questions we posed:

- Is it a good idea to celebrate a National Day of Mathematics?
- Do you think this celebration impacts on the routine teaching of mathematics over the year?
- Do you have any personal recollections and/or anecdotes connected to such celebrations?
- Do you have any suggestions for a more meaningful, impactful celebration?

We received responses from several teachers, teacher educators and practitioners in the field of mathematics pedagogy, whose profiles are given at the end of the article. The majority felt that a special day focused on Mathematics was definitely a good idea and one that had the potential to light sparks of interest in the most reluctant of students.

Often school / college mathematics is directed towards the exam and mired in routines. So, any reason to break the routine and to create an opportunity to introduce students to a different facet of Maths, that is distant from the exam mode is always welcome, if not essential, says **Jayasree Subramanian**.

Sowmyashree N J shared *I believe it is a good idea to have a National Day of Mathematics. It is also an opportunity to remove the fear of mathematics, which is so common among students, and instead allow them to experience joy, discovery, and purpose in the subject.*

But **Ashish Gupta** has a word of caution. *Merely celebrating a day, like we celebrate festivals, does not by itself bring lasting change. For example, celebrating Diwali on one day does not mean that good has triumphed over evil forever. Similarly, observing National Mathematics Day alone will not transform the reality of mathematics classrooms in our schools. What truly matters is understanding the purpose behind such observances.*

Ashish goes on to explain when such a celebration can have a meaningful and long-lasting impact. *The larger purpose of celebrating National Mathematics Day is to inspire children to explore the world of mathematics and to motivate teachers to make their teaching more engaging and enjoyable, so that no child grows up fearing the subject. The celebration also serves as an opportunity to promote mathematical thinking and problem-solving skills among students and the wider community. By dedicating a day to mathematics and to a mathematician like Ramanujan, we acknowledge the subject's*

Keywords: Mathematics day, activities, puzzles, exploration, discovery.

importance in national progress as well as in the everyday lives of individuals. The day also reaffirms India's rich mathematical heritage—from ancient contributions such as the concept of zero and the decimal system to modern-day advancements. It reminds us that mathematics is both a cultural treasure and a vital tool for shaping the future. Equally, it encourages teachers to take initiatives that nurture analytical skills, logical reasoning, and creativity in young learners by influencing their regular classroom practices. However, this transformation cannot be achieved in a single day. It must become an integral part of everyday teaching and learning in all mathematics classrooms.

Nujahat Anjum shares, *A lot of questions arise in the minds of our teachers when we hear these words*

- *Will children who do not learn even if they study for a whole year learn from one day?*
- *We don't have time to teach, where do we get time to do these kinds of rituals?*
- *We don't have learning, how should we go about this?*
- *I am not a subject teacher, why should I?*
- *Our children have no knowledge of numbers, no basic functions, what should we do?*
- *If there are no other teachers in our school who think about it, why should I think alone?*

*The answer I found to these questions was that **I am a teacher**, so:*

- *I try different ways to improve the learning of children, one of which is Mathematics Day.*
- *I make time for children to play and learn with and from different teaching-learning materials.*
- *My job is to teach children numeracy knowledge, basic functions, and I try to do this by routine and non-routine methods that inculcate this knowledge in children.*
- *I teach by example, and I learn when I try new things.*

As **Kanchan** says, *This celebration, when done with purpose, helps in teaching mathematics by giving many teachers a chance to reflect on how their students learn and where they face challenges. Often, after such events, classrooms become richer in their collection of math materials. In many schools, the real hero in this process is the mathematics kit. In both situations, whether teachers and students are already curious or the celebration is held just for the sake of it, I have seen this kit give both students and teachers something new to explore and reflect upon, bringing concepts to life in the process.*

Observation: We need to have a clear understanding of the objectives of celebrating the National Day of Mathematics.

Snapshots of celebrations across India

Jayasree: *One very vivid recollection I have is the celebration of Ramanujan's birth centenary in my college in small town Palakkad where I grew up – I was in Class 11 then. My college organised a quiz competition and some talks on the life and work of Ramanujan. I was not very aware of Ramanujan's work at that time, I am not sure if I had even heard the name. At that time, I didn't think that there could be a "Maths Quiz". How could one be "quizzed" in Maths? I remember participating in the quiz and getting a feel for Maths outside the textbook. The word "mathematician" acquired a different meaning for me. Till then "mathematicians" were names I encountered in textbooks – Pythagoras, Euclid- Kumbakonam and Chennai on the other hand were real places for me – places which I had been* *to. That a "mathematician" could be from these places, and that the schools and colleges he attended still existed was something for me. "Mathematician" was not such a distant notion after all!*

Later in life, in another small town in Gujarat where my children grew up and when many of the people I connected with were 2D beings on the computer screen, I remember the craze with which my children solved problems online during "Pi Day". There was some online contest which the school had shared with all students and the feverish energy with which a whole bunch of kids worked on routine exercises (arithmetic calculations) to have their school name in the "leaderboard" for having solved the most number of problems was amazing! This happened for 2- 3 years. I am not advocating the "competitive spirit" or the meaningless calculations here – but want to draw attention to the "sense of mission" with which the kids went about it. If a "celebration mode" could channelise that energy into more engaging activities that would generate interest in mathematics, that would be great.

I think the "impact" lies in being able to reach small towns and villages. I also appreciate the IMU (International Mathematics Union) "Pi Day" Celebrations, where there is a theme for the activities, where you can share your little celebration with others, get to see how others celebrated and enrich yourself.

A practical suggestion from **G. Jagadeesha**: *If it's celebrated at the cluster level, with all schools in taluk places participating, it will be very effective.*

Here is an account from **Karan Singh**: *Last year, from 17–20 December 2024, we organised a four-day Mathematics TLM-making workshop with 35 upper primary school teachers in Rudraprayag. Teachers prepared individual TLMs on different mathematical concepts and later showcased them at a Teachers' Mela in DIET Rudraprayag. Nearby UPS, GIC students visited the mela, interacted with the teachers, and asked about the use of each TLM. Teachers explained the concepts in detail, making it a rich learning experience for the children. Finally, we encouraged teachers to celebrate the National Day of Mathematics on 22 December in their respective schools and share photos and videos of the event. This initiative was very well received and implemented meaningfully in the field. The Government of Uttarakhand even issued an advisory to all schools to celebrate the National Day of Mathematics. Along with this, we shared ideas, suggested activities, and possible materials for the celebration. It turned into a day where mathematics learning happened with joy, creativity, and freedom, setting a positive tone for teaching throughout the year.*

Pooja Dumaga says *I recall a Maths DRG workshop conducted in Pauri with selected DRG members, which focused entirely on the use of Teaching-Learning Materials (TLMs) in elementary grades in the context of the National Day of Mathematics. As a follow-up, meetings were organised where 3-4 teachers brought the TLMs they had developed and shared their experiences with other teachers. A teacher explained how she used a number line scale to teach addition and subtraction of integers effectively. Another demonstrated how he introduced the concepts of $\sqrt{2}$ and $\sqrt{3}$ using a practical tool. After the workshop, 7-8 teachers shared photos and videos of the Maths Corners they had created in their schools. Some also began exploring mathematical identities using cut-outs.*

Observation: A National Mathematics Day celebration is particularly relevant in remote areas. Schools in such areas can do joint celebrations which pool resources and serve as a platform for exchanging learning.

Suggestions for a meaningful celebration:

Pooja: *From the above experiences, I learned that instead of talking about many different TLMs, it is better to focus on just a few and explain them clearly using proper mathematical language. This helps teachers think in a more complete way, rather than just making TLMs without understanding.*

Pooja reiterates that students should be told about the lives and work of famous mathematicians – this would help students appreciate mathematics and see that it is connected to real people and their stories.

Figure 1: Paper-cutting activity based on the symmetry of shapes

Hands-on activities seem to be the key. **Saddam Husain** says *Last time, paper craft activities were planned to work with children of class 4-5 and upper primary level. The main purpose of these activities was to promote visualization. It has been seen in the field that when interesting activities with children are demonstrated on Mathematics Day, its effect has also been seen in the teaching process of teachers for some time after.*

Mokhtar Zaman: *In our school's Baal Shodh Mela, we had a mathematics corner. Children enjoyed solving puzzles, playing number games, and exploring activities. I saw that the same students later showed more interest in the classroom when we taught them similar topics such as weight in the chapter on measurement, and angles in geometry.*

When a mathematics laboratory was set up in our school, we were told that we could even try out small research activities. At that time, we students were still confused and kept wondering, "How can we research in Maths?" But soon, our curiosity turned into excitement when we were given tasks to create small projects. I still remember choosing the algebraic formula $(a + b)^2 = a^2 + b^2 + 2ab$*. Since some construction work was going on at my home, I used a marble piece and carved out two squares and two rectangles on it to demonstrate the formula. It was a simple idea, but I felt very proud of it. Later, when all the students brought their own models, we organized an exhibition on 22nd December, celebrated as National Mathematics Day. That was the very first time our school celebrated Maths Day, and it started with our own projects and creativity.*

Sowmyashree says *there are many ways of celebrating this day, and I remember one particularly meaningful celebration when I was a teacher. I was given the chance to plan activities with high school students. Together with teachers and parents, we designed an event that gave responsibility to the students themselves.*

Students take the lead - *Students of Grades 8 to 10 took the lead in training and guiding younger children. One group conducted a simple survey on our school road to find out how many students carried plastic bottles to school. They compiled and presented their findings to the whole school assembly. The process of collecting, organising, and sharing data gave them a new sense of mathematical purpose. More importantly, their work had a direct impact: the head teacher decided to observe a "no plastic* *usage" week in the school. This showed students how mathematics could influence real decisions and bring about positive change in their community.*

Learning together *- Another group worked with younger children who often struggled with mathematics. The high school students taught them concepts like area and perimeter in concrete ways—taking them to the playground with measuring tapes, using graph paper to calculate area, and making the ideas come alive. Parents and teachers supported these sessions, and the younger students not only understood better but also felt more confident. Watching older students patiently teach and explain was heartwarming—it made mathematics collaborative, supportive, and joyful.*

We hear from **Narender Kothiyal** of the learning from organizing such celebrations: *Mathematics needs to relate to the day-to-day life experiences, thus assignments need to relate with the application of mathematics.*

Model making is an interest of most of the students. It just needs to be connected meaningfully to the theory that is taught in order to enhance learning. For example, what happens when the radius of one cylinder is increased by a certain value, and the height of a second cylinder is increased by the same value? Then what happens to their volumes? So, such types of things when included gives the students depth to their learning and calculations. Here are a few key ways to ensure this:

a. Students must be prepared for questions related to the topic they are doing so that they can also deal with the questions more confidently.

b. More opportunities need to be given for independent exploration and understanding.

c. Students need to practise and rehearse for the event.

d. Proper time needs to be given for preparation.

Kanchan sums up the discussion: *Celebrating the National Day of Mathematics is good but it will be meaningful only when it goes beyond being just a formality. Its real goal is to create an environment where students feel encouraged to explore, ask questions, make mistakes and enjoy learning math. I have witnessed both types of celebrations. In one primary classroom, children thought, played games, faced challenges, and even made mistakes while solving puzzles and recognizing patterns. In another classroom, the event felt more like a formality, lacking the connection to its purpose. However, in both situations, the positive outcome is that these events made teachers and students step out of their usual routines and provide something new to think upon. Designing problems, puzzles and games based on Learning Outcomes connected to the content already taught serves as a fun, low-pressure assessment tool for teachers to see how well students have understood concepts while letting students enjoy things.*

Observation: Activities must connect to the curriculum and help students to view content in textbooks not just through pen and paper exercises.

The day can serve as an opportunity for informal assessment, not just for the students but also for the efficacy of the teaching.

When students are given the responsibility, they learn much more than mathematics, this is an opportunity to develop life-skills. This celebration is an opportunity to inspire children: with stories about mathematicians and their relentless efforts, with examples of applications of mathematics, with the sheer joy of doing mathematics.

Keeping in mind the learning outcomes for each class, while organising such days, the vision for teaching school mathematics does not remain a distant dream but an actual possibility.

Conclusion

It was heart-warming to see the level of thought and preparation that goes into the celebration of Mathematics Day across the country. It is an opportunity to bring the curriculum alive and make learning exciting, provided that there is a clear understanding of the objectives of such a celebration. It provides opportunities for students with different abilities to excel and can build individual and personal capabilities and confidence.

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The National Day of Mathematics

Planning meaningful celebrations in schools

Padmapriya Shirali

In my school days we used to participate occasionally in science and mathematics exhibitions. While there was a fun element to the whole experience, we did not experience the richness of a well-organised exhibition. Since we were assigned stalls to man, we often spent the whole day there and did not make use of the opportunity to view other stalls, engage with others and reflect on the exhibits. My experiences as a student and later as a teacher made me realise that designing a well-structured mathematics exhibition or celebrating a mathematics day calls for careful planning.

During my teaching career at Rishi Valley, I got introduced to the idea of a Mathematics Exposition (Math Expo) under the guidance of Mr. P.K.Srinivasan. I continued this practice for many years, working along with other members of the mathematics faculty.

My mentor's vision of a Math Expo involved a 'walk through the curriculum'. The topics selected were based on concepts of school mathematics and presentations were made by using materials, cards, posters, math games, puzzles, etc. The expo had a dual purpose. For every student, as they visited each stall, the presentation either served as a review of concepts learnt in lower grades or as a window into the concepts that they would come across in the higher grades. As the concepts were revisited in the form of games or with physical aids, they held the interest of the visitors. Of course, it stimulated interest in the subject in general and gave the students an opportunity to talk about mathematics, subtly boosting their confidence and interest in the subject.

In India, National Mathematics Day is celebrated on December 22, to commemorate the birth anniversary of the renowned mathematician Srinivasa Ramanujan. The International Day of Mathematics is celebrated on March 14^{th}, because it represents the first three digits of Pi (3.14) when written in MM/DD format.

Many schools celebrate a mathematics day and have multiple engaging activities and stimulating presentations that are normally not taken up during teaching sessions due to constraints of time.

I will briefly dwell on the organisational aspects of such a day. The pullout of this (November 2025) issue carries some ideas for activities for such a day. It is always good to have various strands represented in the activities - Arithmetic, Geometry, Statistics, Logic, etc. It is also necessary to remember that a good proportion of activities need to be accessible to a layman so that parents and visitors are motivated to take part in it. We work on the principle, 'Math for All!'

Keywords: Mathematics exposition, festival, logistics, arrangements, preparation

As the visitors to the stalls will be from diverse backgrounds, the presentation should be adaptable to the varied levels to the extent possible. The activities should also lend themselves to a limited time interval (5 to 10 minutes) per stall to accommodate all the visitors.

Organisation of the student groups

A group of 4 students can explore the theme to be presented at the stall. Groups can be formed class-wise so that every student from the school is part of some group. Two students from each group would, at a time, get involved in demonstrating the topic they had chosen. During that time, the other two get the opportunity to walk around the other stalls till it is time for them to man their stall. This will give everyone a chance to observe all the stalls.

What is the preparation required by the students for such an event?

Students need to work on the problem they are going to present and arrive at one or multiple solutions. They should anticipate how the audience might respond and be prepared to address a variety of responses. They should be able to explain the solution if needed. This will need some practice beforehand. Possible criteria for the selection of problems, which can be done by the teacher or by the students themselves, are: can be solved in 5-10 minutes, can have multi-levels so that it can be attempted by an audience with a spectrum of ability and knowledge of mathematics, can be an extension of what is done in class, and so on.

Layout of the stalls

It is good to use a large area such as the school playground (outdoor venue) or the assembly hall (indoor venue). A group of 4 or 5 stalls can be clustered together, separated from another cluster by some distance. This will help in containing the noise generated by the chatter and excitement. Within each group of stalls, I prefer to have separate stalls for number-based, geometry, estimation and mathematics through visuals/patterns activities. Clubbing all stalls related to a particular strand together, can tend to be repetitive and may lead to overcrowding in some areas.

A few games can be interspersed between the different clusters to serve as buffer spaces if a group of visitors needs to wait for some time before they are able to move on to the next area.

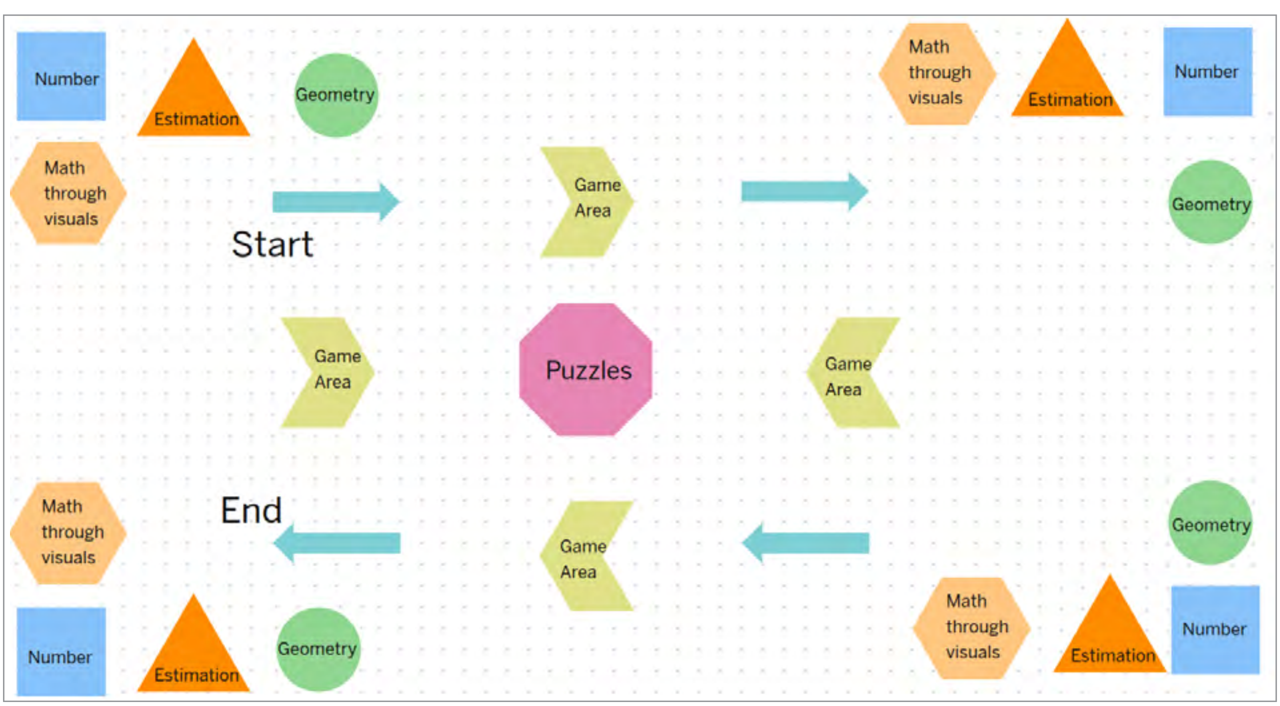


Figure 1: A suggested configuration for the stalls at a Math Expo.

The organisational aspects of such an event in itself involves mathematical planning – estimating the number of visitors, qualitative time required to engage with all the stalls and the sequence of movements. It will be a nice exercise for the senior students to come up with a good route plan for groups of individuals to move from one area to another so that overcrowding does not occur in some areas. Safety precautions such as a quick evacuation plan in case of an emergency should also be factored in by them.

Organisation of the day's schedule

Schools start the day with an assembly time when all the students and staff come together to share music, news and other information. The assembly at the celebration of a Mathematics day can focus on interesting facts about that date, or the work of a particular mathematician. Students present the dramatisation of short skits based on interesting anecdotes from the life of a mathematician, or on math themed stories at the assembly. Researching and writing such scripts and stories will develop key inter-disciplinary skills. Students can also talk about recent discoveries or unsolved problems in mathematics. They can share fast calculation tricks that they have learnt or talk about how they feel about the subject, what interests them and why they would like to pursue their study of it. Video clips that show symmetry, animation of 3D shapes, patterns, illusions, etc., can be shown.

Students who don the role of presenter or facilitator at a mathematics event often carry the glow of being in charge into the classroom. It is hoped that they will realise that they are active participants in the process of learning mathematics.

Note from the Editor: These suggestions give an outline plan for a meaningful and orderly celebration of a Mathematics Day which has the potential to draw in even the most reluctant student of the subject. Details of possible activities, filler space games and cultural events are given in the Pullout.

Padmapriya Shirali is part of the Community Math Centre based in Valley School (Bangalore) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. In the 1990s, she worked closely with the late Shri P K Srinivasan. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box.' She is currently part of the NCERT textbook development group. Padmapriya may be contacted at padmapriya.shirali@gmail.com

Proposed solution for the Art in Numerals puzzle in At Right Angles July 2025 (page 8)

Solutions to the Art in Numerals Puzzle published on page 8 of At Right Angles, July 2025 provided by **Karthikeyan SS**, working as Resource Person at Azim Premji Foundation Puducherry District Institute.

Q1: Can you find a pattern connecting these numbers?

3×2	3×3	3×4
3×9	3×10	3×11
3×16	3×17	3×18
3×23	3×24	3×25

General Observation:

Basically, this is the multiplication table of 3 arranged in a grid

Q2: Can you find such a pattern in a different set of numbers?

2 *n*	**3 *n***	**4 *n***
9 *n*	**10 *n***	**11 *n***
16 *n*	**17 *n***	**18 *n***
23 *n*	**24 *n***	**25 *n***

where n is a natural number.

Example:

$$\frac{(3n + 9n + 11n + 17n)}{4} = 10\,\boldsymbol{n}$$

Both the diamond pattern and the rectangle pattern work in this grid.

What We Steal When We Teach

Smruti Smarak Panda

"The scandal of education is that every time you teach something, you deprive a child of the pleasure and benefit of discovery."
– Seymour Papert, The Children's Machine (1993)

About 95 years ago, a two-year-old was obsessed with automobiles. He loved cars so much that he could name all the parts of the car. Over time, he understood how gears work, and then became so deeply involved with gears that they became one of his favourite toys. He liked rotating circular objects such as bottle caps against each other. It was fascinating to see how turning one of the gears in one direction rotated the other gear in a different direction. That was his first interaction with, and understanding of, chains of cause and effect.

Figure 1: Source: https://bit.ly/4ovZAA0

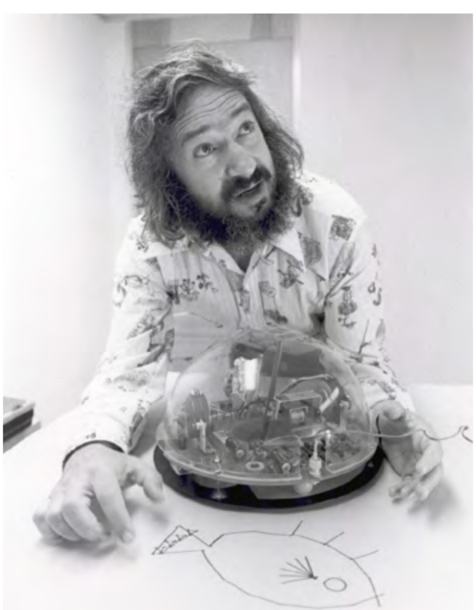


Figure 2: Source: https://bit.ly/43yokzx

That man, that legend - that history fondly remembers was a very special person. He was Seymour Papert. He believed that if he described his play with most educationalists, they'd recommend that he create a gear set which children could use to learn about gears. But the essence of his story, was that he loved gears. That is why he could play endlessly with them. He liked rotating gears - and saw the manual that came with such a gear set as an interference.

After consistently working on his ideas on pedagogy, he reached a point where he concluded that while a gear set without a rigid set of instructions might not be able to promote exploration and discovery, perhaps computers could play this role. He developed the game LOGO (see [2]) and developed it to fill what he felt was the need of the hour.

Keywords: Discovery, exploration, independent learning, scaffolding

I had never heard of Papert until I came across his book Mindstorms [1], a while ago. In Mindstorms, he mentions his interaction with gears in the preface. From there, he argues that computers and programming can serve a similar role for children, creating environments where mathematical ideas feel natural rather than imposed. His central belief is that learning is not about consuming knowledge but about living within an environment that supports it. Just as a child growing up in England learns fluent English simply by being immersed in an English-speaking environment, while the same child might struggle with conversing in English in India. Papert suggests that schools can use computers to build mathematical “environments” where young kids construct knowledge naturally, through exploration and visual programming, rather than treating mathematics as an alien and difficult subject.

Democracy within the Classroom

Now, shedding some light on my recent endeavours of volunteering to teach English at a local Odia Medium Primary School. All of the class environment restructuring that I will speak of in this segment, might not have any direct connection with mathematics learning, but then, neither did Papert’s gears. The idea, for me, is to establish the kind of environment I wanted the classroom to shape into. A classroom where knowledge and learning were constructed by them and they felt a sense of ownership over their learning. We would only provide prompts where necessary, to scaffold their learning. This keeps the students excited and engaged. Their brains are consistently made to think unconventionally rather than memorize information and write answers. Many students develop a fear of mathematics even before engaging with it, as their environment repeatedly reinforces the belief that the subject is inherently difficult and intimidating and that there is only one correct process which would lead to one correct answer. So, it felt more natural to first get them to think and talk rather than throwing problems in mathematics at them.

Inspired by Robin Williams’ Mr. Keatings from the movie *Dead Poets Society*, I was determined to build a classroom that learnt to think and feel deeply and seize every day. In Class 5, I decided to build a class reflecting democracy. First, we formed a cabinet of ministers. Pragyan was Chief Minister, Ipsita the Reading Minister. When students hesitated to read aloud, they divided the class into two groups and led their peers. The room turned noisy but joyful, full of reading and peer support. Later, we expanded into a Legislative Body: Class 5 as Lok Sabha, Class 4 as Rajya Sabha, with teachers as presiding officers. A debate on whether cinema was good or bad ended with a unanimous vote in favour of films as a source of knowledge and exposure. A detailed account of the results we observed during these proceedings is a little beyond the scope of this article which shall primarily focus on the mathematical experiences.

The Problem of the Day

Two young students named Sai Nigam and Swagat, who loved mathematics a lot were appointed as the Problem of the Day Ministers. Their job was to write one interesting problem a day on an unused blackboard of the school. Other students could read and solve these during the school hours. It was decided that the solution of the problem would be discussed at the end of school hours every day.

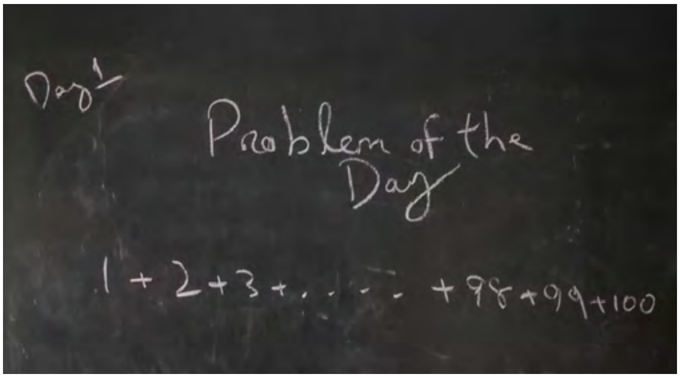


Figure 3

The two 'ministers' were provided with an Odia book named "Dinaku Khandie Anka (One Problem a Day)" authored by Prof. Chandra Kishore Mahapatra who in his prime was an active Olympiad Trainer in Odisha. The book had a very interesting structure. There were 12 chapters each representing a month, and each chapter consisted of 30 or 31 or 28 questions depending on the month after which the chapter was named. The solutions to all the questions, which were comprehensible with self-study, were placed after the problems chapters. But I had made it clear that we should try a question for at least 30 minutes to 1 hour before we decide to look into the solutions. An important consideration by me for selecting the specific students for the Problem of the Day Minister role was based on their honesty and sincerity. The first problem that was written on the board on July 23, 2025 was the famous 1 + 2 + 3 + 4 …… + 97 + 98 + 99 + 100 = ? which was taken from the cover of another book by Prof C.K. Mahapatra.

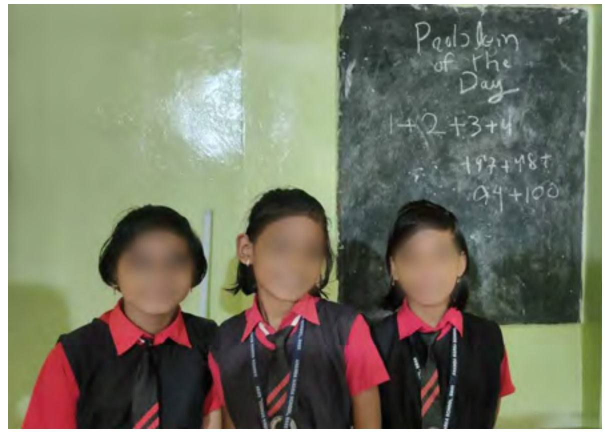


Figure 4

Now a majority of the students ruled out the possibility of this question being solved. But three students named Sai Sampurna, Pragyan and Ipsita were actively working on the question. More importantly they were working on it together. Exactly the kind of collaboration that I had been hoping would be a consequence of the Democratic classroom design. Even during the recess, I could see them solving it.

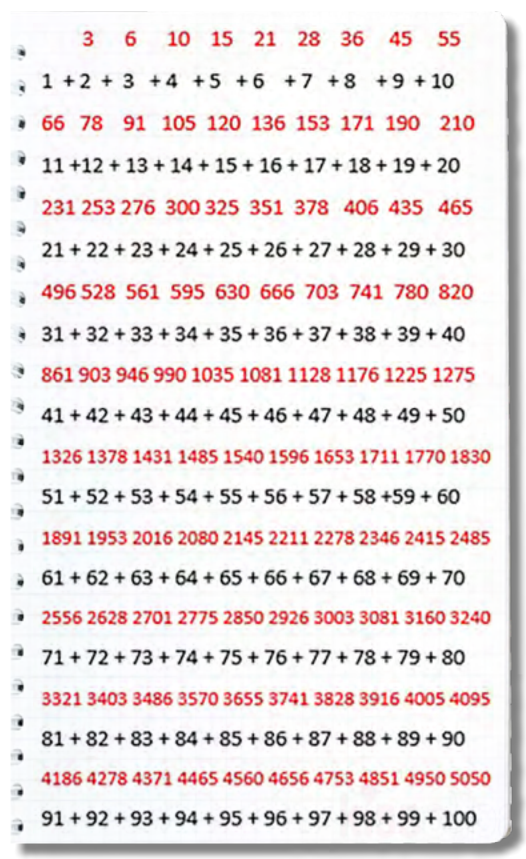
1 + 2 = 3

3 + 3 = 6

6 + 4 = 10

10 + 5 = 15

And so on until they just got confused and ended up adding a number twice or just getting the wrong sum and feeling the need to start over again. Even the other teachers of the school saw them attempt this question repeatedly. About an hour later, when I was teaching in Class 4, these three students came up with the answer 5050 and more importantly the solution written on a page which has been replicated in Figure 5 for the benefit of the reader.



3 6 10 15 21 28 36 45 55
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10
66 78 91 105 120 136 153 171 190 210
11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20
231 253 276 300 325 351 378 406 435 465
21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29 + 30
496 528 561 595 630 666 703 741 780 820
31 + 32 + 33 + 34 + 35 + 36 + 37 + 38 + 39 + 40
861 903 946 990 1035 1081 1128 1176 1225 1275
41 + 42 + 43 + 44 + 45 + 46 + 47 + 48 + 49 + 50
1326 1378 1431 1485 1540 1596 1653 1711 1770 1830
51 + 52 + 53 + 54 + 55 + 56 + 57 + 58 + 59 + 60
1891 1953 2016 2080 2145 2211 2278 2346 2415 2485
61 + 62 + 63 + 64 + 65 + 66 + 67 + 68 + 69 + 70
2556 2628 2701 2775 2850 2926 3003 3081 3160 3240
71 + 72 + 73 + 74 + 75 + 76 + 77 + 78 + 79 + 80
3321 3403 3486 3570 3655 3741 3828 3916 4005 4095
81 + 82 + 83 + 84 + 85 + 86 + 87 + 88 + 89 + 90
4186 4278 4371 4465 4560 4656 4753 4851 4950 5050
91 + 92 + 93 + 94 + 95 + 96 + 97 + 98 + 99 + 100

Figure 5

One can clearly see the effort and the structure in approaching this problem that these three students took to reach the solution. Are they as good as Carl Friedrich Gauss who devised a very popular solution to this question during his school years? Well, no, at least not yet. But are they better than students who waited to be taught a method or were taught the Gauss Method without even taking a chance to solve this question by themselves? I believe yes.

Figure 6

Of course, I told them and the others in the classroom the story of Gauss and his teacher after that and one can imagine how exciting it must've been for some students, who for the last hour were only interested in solving the question. When I showed them the picture of Gauss, I'm sure that was the first time they saw a foreign mathematician/ scientist who wasn't Einstein; I'm sure it was the first time they heard a story around mathematics, that too a real one. After this, I also showed them a YouTube video of the Art of Problem Solving where Richard Rusczyk was solving this very question in a 2:49 minutes timed video. We all proceeded to discover a playlist of 151 Prealgebra videos [6] in the same YouTube channel by going through the recommendations.

A Problem of the Day ritual followed in a classroom can set a lovely atmosphere of mathematical thinking. These problems can be of a wide variety and can be taken from age-appropriate mathematics contests from around the world. (I've put the link to some such contests in the references.) It helps to give problems where they can take their time and figure out various ways of solving it. For example, in a problem like the one shown in Figure 7, one could simply solve this question if they notice that the number of 1s and 0s in each column is the same and is equal to 2. But on the other hand, a group of students might simply sit down and make an entire diagram of the fixtures, which can be a really creative outcome. It also helps you follow up the momentum by designing questions made on fixtures and tournaments. The kind of problems that we need to avoid are problems which are too difficult, because that will have them lose both interest and confidence. If they continue solving these non-routine easy problems, it's only a matter of time before they learn to figure out difficult problems, which generally are a culmination of two or more simple problem-solving thoughts.

Lola, Lolo, Tiya, and Tiyo participated in a ping pong tournament. Each player competed against each of the other three players exactly twice. Shown below are the win-loss records for the players. The numbers 1 and 0 represent a win or loss, respectively. For example, Lola won five matches and lost the fourth match. What was Tiyo's win-loss record?

Player	Result
Lola	111011
Lolo	101010
Tiya	010100
Tiyo	??????

A) 000101 B) 001001 C) 010000 D) 010101 E) 011000

Figure 7: 2023 AMC 8 Problems/Problem 8

Having said that, it is preferable to take problems that can expand into some kind of pattern, or a problem that has more than one answer. For instance, ask them to add the first n consecutive odd numbers, giving them the question for very small values of n. Encourage them to look for patterns in the answer. When they realise that all the sums are square numbers, slowly guide them, by using Chess Board with pieces or Four in a Row Board or any such game which you find available. It will take them time, but with some hints, they'd remember the experience of figuring out the geometric representation lifelong. These kinds of problems are available in plenty. For starters, one can find such problems in the Pull-Out section of At Right Angles, March 2025 issue, a highly enriching article named Patterns and Pre-Algebra by Padmapriya Shirali.

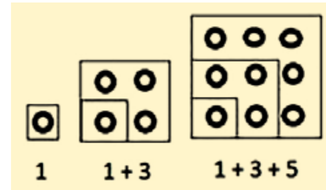


Figure 8

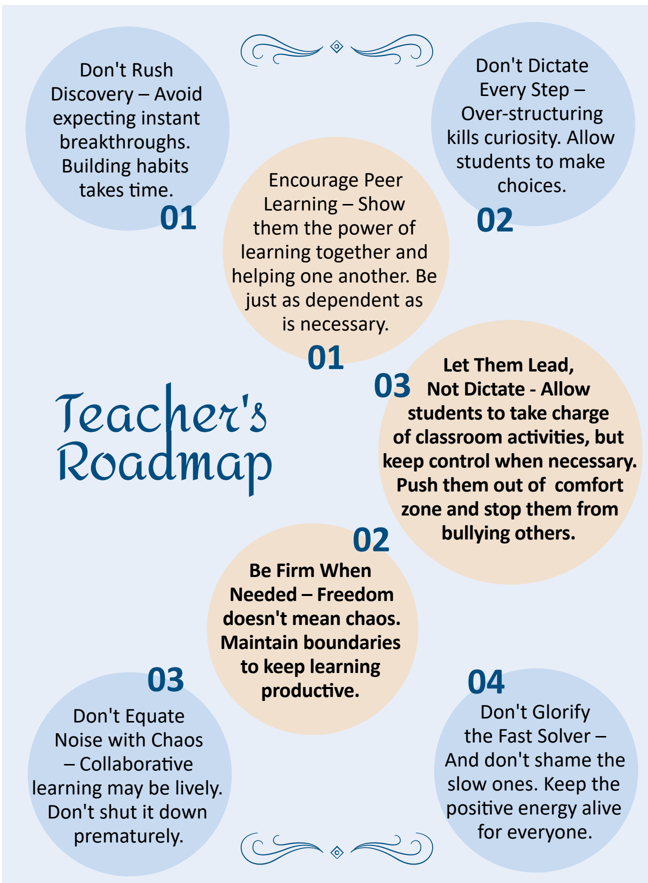


Figure 9

Final Advice

Discovery-based learning proceeds slowly initially but can accelerate exponentially. Even the anecdote I've shared about the problem came after over a month of classes in April vacation and a few days after school reopened post the summer break where we were focusing entirely on Language and Social Studies to build their thought process before diving into mathematics. The goal is to help students learn how to learn: how to navigate the internet, follow meaningful YouTube playlists, and take notes from videos. By the time they're in Class 8, who knows, they might bring you a write-up they want to submit to a mathematics magazine like *At Right Angles*. After all, in the very words of Seymour Papert:

> "The role of the teacher is to create the conditions for invention rather than provide ready-made knowledge."

That happened to me. My father, a physicist himself, encouraged me to explore the internet thoughtfully, told me stories from Sherlock Holmes and watched cinema of the likes of Bicycle Thieves with me. With time, I found myself writing short detective stories around cryptography and codes and submitting them to magazines like Stone Soup. I never got published, but looking back it's among the very innocent and enriching memories I have. The real task is to take children seriously, not as passive learners but as active builders of their own knowledge. The greater task is not just to teach but to slowly teach them how to teach themselves.

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A Call for Participation

Mathematics, by nature, is cumulative and hierarchical. Therefore, from a teacher's perspective, it is important to ensure each time that the learners have no gaps in their prerequisite knowledge. In some cases, this may require recall or revision of concepts taught in previous grades. We, at At Right Angles, are conceiving the idea of bringing out articles that discuss strategies employed by teachers in introducing new topics. We will keep in mind the prerequisites, as well as, certain challenges they may have faced when prerequisites are not achieved by a significant proportion of the class. We will also discuss how they could navigate through such challenges.

For instance, the topic 'Fractions' is now introduced in grade 3 and continues in grades 4 and 5. One can identify the following prerequisites for the chapter on fractions in grade 3:

- **Broadly** - number sense, operations of addition and multiplication by 2 on natural numbers
- **Broadly** - intuitive sense of area - comparing them to conclude which is more, which is less, which are equal
- **Specifically** - comparing natural numbers (at least smaller numbers)

The teacher has to notice the prerequisites in terms of concepts, skills, thinking (multiplicative thinking/additive thinking), computations, and sometimes intuitive understanding needed for a topic.

We request our readers, particularly teachers engaging students of grades 3, 4 and 5 to take time in filling out the form (QR code & link provided https://bit.ly/49dSbkn) to help us understand how one has gone about these processes during the planning for and teaching of new topics.

Little Childhood Mathematical Discoveries about Divisibility

Jeenath Rahaman

Inspired by the story behind Chika's test for divisibility by 7 in the March 2020 issue of *At Right Angles*, I wanted to share a similar story, which doesn't stop at finding the test for divisibility by 7, but goes on to discover the tests for divisibility by any number. I hope this story inspires young learners and even adults to keep making these little discoveries and to share the joy with others.

NCERT (2017) [1] recommends observing patterns that lead to divisibility by 2, 3, 4, 5, 6, 9, 10 and 11 in the suggested pedagogical processes (p. 67) for Class 6. As rightly pointed out in the article mentioned above, the tests for divisibility by such numbers are popular among school students. Care must be taken to ensure that students discover and appreciate these interesting number patterns instead of practising them as prescribed rules. Chika's test for divisibility by 7 was to multiply the units digit of a given number by 5 and add it to the remaining (truncated) part of the number. The original number will be a multiple of 7 if and only if the resulting number is a multiple of 7. However, in this article, we will learn about an investigation which will allow us to create several simpler tests for divisibility by 7 and other divisors, for example 13, 17, 19, 23 and so on. These tests are not widely known.

Encouraging students to notice these divisibility tests will enrich students' perceptions about numbers and develop their number sense. This will allow students to explore and later reason out interesting patterns with numbers which can further contribute to students' mathematical thinking. Another common aspect between this article and Chika's story is the role of mathematical intuition in discovering interesting patterns while seeing relations between numbers and operations.

We begin with the story of a young girl called Maya, whose favourite pastime was to discover new divisibility tests for any number. Once we understand her set of simple tricks, we will be able to create our own general rules to check for the divisibility by any number without going into any complex mathematics or memorizing any rule. Wouldn't that be fun? But to know this trick, you need to know Maya's modus operandi. For our convenience, we will define and use some pre-defined symbols. We will denote the units digit of the given number by U and the truncated number after removing the units digits by T. For example, if the number is 5382, then U will be 2 and T will be 538, and if the number is 394, then U will be 4 and T will be 39, and so on.

To begin with, to find the divisibility test for 7, Maya would first write down the multiples of 7, i.e., 7, 14, 21, 28 and so on. She would then think of an operation between the digits of the multiples of 7 to get the outcome as 0 or 7. Thus, taking a cue from the second and fourth multiples of 7 (i.e., 14, 28), her trick would be to multiply the tens digit by 4, and find the difference between this product and the units digit to get the resulting value as 0 or 7 (Table 1).

Keywords: Divisibility, patterns, exploration, verification

Table 1. Trick-1: (T × 4) – U. (U = units digit, T = truncated number).

The multiples 14 and 28 helped Maya make this rule.		
Repeat until the result is 0 or 7.		
Verifying the trick for a 2-digit number 84 (U = 4, T = 8)	Verifying the trick for a 3-digit number 959 (U = 9, T = 95)	Verifying the trick for a 4-digit number 9261 (U = 1, T = 926)
8 × 4 – 4 = 28 Repeat for 28 U = 8, T = 2 2 × 4 – 8 = 0	95 × 4 – 9 = 371 Repeat for 371 (U = 1, T = 37) 37 × 4 – 1 = 147 Repeat for 147 (U = 7, T =14) 14 × 4 – 7 = 49 Repeat for 49 (U = 9, T = 4) 4 × 4 – 9 = 7	926 × 4 – 1 = 3703 For 3703 (U = 3, T = 370) 370 × 4 – 3 = 1477 For 1477 (U = 7, T = 147) 147 × 4 – 7 = 581 For 581 (U = 1, T = 58) 58 × 4 – 1 = 231 For 231 (U = 1, T = 23) 23 × 4 – 1 = 91 For 91 (U = 1, T = 9) 9 × 4 – 1 = 35 For 35 (U=5, T = 3) 3 × 4 – 5 = 7

The most wonderful part about this trick is that it can be used as a test of divisibility by 7 for any multiple of 7. This trick can be proved using algebra or modulo arithmetic. However, as this is beyond the scope of elementary school mathematics, the proof is not discussed in this article.

Though the operations have been repeated until the result is 0 or 7, the process may be stopped and a conclusion reached when the result is a recognisable multiple of 7. For example, in columns 2 and 3 of Table 1, the process can be stopped on arriving at 147 and 1477, respectively.

What if there is a more efficient trick, which can reduce the number of digits in a number with each succeeding step? Maya thought of a new trick, sparked by the digits of the specific multiple of 7, i.e., 21.

Trick-2 was to multiply the units digit with 2 and find the difference between this product and the tens digit, i.e., T – (2 × U), to get the resulting number as 0 or 7 (as shown in Table 2). As before, this trick can be proved to be true for all multiples of 7.

Table 2. Trick-2: T – (2 × U). (U = units digit, T = truncated number).

The multiples 21 and 42 helped Maya make this rule.		
Repeat until the result is 0 or 7 or a known multiple of 7.		
Verifying the trick for a 2-digit number 84 (U = 4, T = 8)	Verifying the trick for a 3-digit number 959 (U = 9, T = 95)	Verifying the trick for a 4-digit number 9261 (U = 1, T = 926)
8 – (2 × 4) = 0	95 – (2 x 9) = 77	926 – (2 x 1) = 924 For 924 (U = 4, T = 92) 92 – (2 x 4) = 84 For 84 (U = 4, T = 8) 8 – (2 x 4) = 0

It is interesting to notice that Trick-2 is more efficient than Trick-1 as Trick-2 is directly reducing a 4-digit number to a 3-digit and then a 3-digit to a 2-digit number to quickly decide if the given number is divisible by 7 or not.

The clue for deriving these tricks is to get 0 or a multiple of 7, with some operations between the units digit and the tens digit observed in two-digit multiples of 7. Chika's test can also be stated as T + (5 × U) using multiples of 7 such as 42 or 35. Further, it would be interesting to check if the above tricks will be more efficient than Chika's test, but this task I leave for the reader. A word of caution, in case students have not encountered negative numbers, the teacher can encourage them to just find the difference between numbers (finding the magnitude and ignoring the sign) while using the trick.

Now, let us explore the divisibility trick for 13. Here again, we will start with writing down the multiples of 13, i.e., 13, 26, 39, 52 and so on. And in this case as well, the approach is to look for some operations between the digits that will lead to 0. In case of multiples of 13, it will be obvious that multiplying the tens digit with 3 and finding the difference between this and the units digit will give 0.

Table 3. Trick-3: T × 3 – U. (U = units digit, T = truncated number)

The multiples 13 and 26 sparked this rule.	
Repeat until the result is 0 or 13 or a recognisable multiple of 13.	
Verifying the tricks for a 3-digit number e.g. 741 (U = 1, T = 74)	Verifying the tricks for a 4-digit number e.g. 3003 (U=3, T =300)
74 × 3 – 1 = 221 For 221 (U = 1, T = 22) 22 × 3 – 1 = 65 For 65 (U = 5, T = 6) 6 × 3 – 5 = 13	300 × 3 – 3 = 897 For 897 (U = 7, T = 89) 89 × 3 – 7 = 260

Here is a task: Does T + 4 × U (sparked by 13, 91 and 52) work to identify multiples of 13? Which of these two tricks do you think works more efficiently, giving you smaller numbers at each step?

Similarly, the test for divisibility by 17 can also be figured out by first writing down its multiples, i.e., 17, 34, and 51 and examining their digits to figure out some pattern. For example, (7 × T) – U (sparked by 17) and T - (5 × U) (sparked by 51) are divisibility tricks that help to identify multiples of 17.

This indicates there is no single divisibility test for a number and that these tests can be derived following these simple rules. In this article, the examples were tried mostly with prime divisors, i.e., the divisors were numbers which have no factor other than 1 and themselves. The reader is encouraged to find more tricks for other prime numbers from the list of primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97 and so on.

Further, one can create these rules even for composite numbers. And I encourage you to try finding out the divisibility test(s) for any other two-digit number of your choice.

It is interesting to notice from this article that the digits of the multiples of a certain number are related by different patterns that lead to divisibility rules. And that all these patterns work for all multiples of the numbers, as seen in the examples discussed in the article.

While the examples used have all been multiples of the divisor under consideration, it would be useful for students to verify that these rules are not satisfied for non-multiples of this number. Such investigations are a subtle opportunity for making practice with number operations interesting. Students feel like investigators and explorers when the mathematics class offers them such avenues. I hope this article revives such intuitive discoveries among learners interested in mathematics.

Acknowledgement: The author is grateful to Aaloka Kanhere and Rossi D'Souza for reviewing the initial draft.

Reference

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Interpretation of the 'Art in Numerals' in page no 8 of the July 2025 issue

6	9	12	15	18	21
27	30	33	36	39	42
48	51	54	57	60	63
69	72	75	78	81	84
90	93	96	99	102	105
111	114	117	120	123	126

In a grid of random numbers, averaging the corner values does not give the middle value. But because the given grid is made by a linear rule (like an arithmetic pattern), this relationship arises. It works not just for rectangles or squares, but for other shapes too as shown.

In both the cases shown, when we add the four numbers in the similarly coloured (yellow or sky blue) cells and divide the sum by 4, we get the number in the centre (red / green cell) of the polygon.

I observed that each step to the right of a number increases the number by 3 and each step down adds 21 to the number. By denoting the number in the $\mathbf{i}^{th}$ row and the $\mathbf{j}^{th}$ column by f_{ij} , I was able to arrive at the general formula that $f_{in} = 21\mathbf{i} + 3\mathbf{j} - 18$. (Start with the assumption that $f_{i,j} = a\mathbf{i} + b\mathbf{j} + c$, where a, b and c are constants and solve simultaneously using specific numbers in the grid.)

This formula gives any number in the grid. (Check: $f_{(1,2)} = 21 \times 1 + 3 \times 2 - 18 = 21 + 6 - 18 = 9$.)

If the number in red in the centre is given by fij , then the numbers in yellow around it are given by $f_{i,j-1}$, f_{i-1},j, $f_{i,j+1}$ and $f_{i+1,j}$. Substituting in the general formula for each of these numbers, we get the number in the centre to be $4 \times f_{ij}$.

We see that as long as the other four numbers are situated symmetrically about the centre number, we will get four times the central number when we add them.

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Exploring Area and Perimeter Through Experience: A Classroom and Cluster-based Journey

Karan Singh

Teaching mathematics in primary schools is often seen as working with numbers, rules, and formulas, but when we apply the mathematics we learn, a world of discovery opens up. This article describes such a journey — working with teachers to explore the concepts of **area and perimeter**, particularly the idea of a **fixed perimeter** and **changing area**. The experiences shared here are rooted in classroom conversations, mistakes, and teacher reflections. This is an experience-based article from four government primary schools and cluster-level teachers' workshops, in Rudraprayag, Uttarakhand.

Measuring pieces of land having different shapes

It began with a discussion during a primary school teachers' cluster meeting in Rudraprayag district. We were discussing Chapter 11 Area and its Boundary in the Grade 5 NCERT Textbook. I made some rectangular shapes each with a fixed perimeter of 44 metres, on the board, as shown in Figure 1.

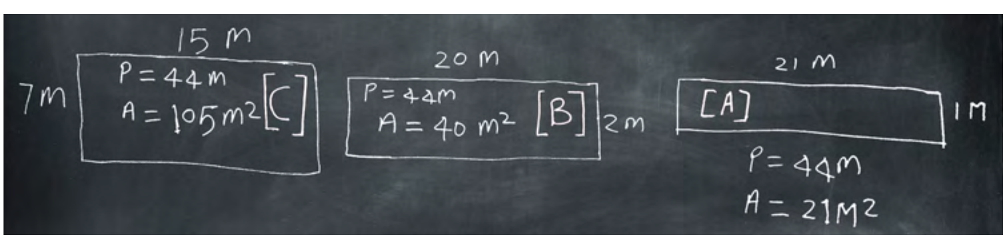


Figure 1

These rectangles immediately caught everyone's attention. Teachers began drawing rectangles with various side lengths — 11 by 11, 12 by 10, 14 by 8, and so on. The calculations followed:

- **11m × 11m (square)** → Area = 121 m^2
- **12m × 10m** → Area = 120 m^2
- **14m × 8m** → Area = 112 m^2

The rectangles had the same **perimeter = 2 × (length + breadth) = 44 m**, but their areas varied.

Then we discussed what happens when the breadth of the rectangle is reduced by x metres and the length is correspondingly increased by x metres (to keep the perimeter constant).

Keywords: Contextual mathematics, conversations, exploration, area, perimeter

Our observation: The maximum area occurred when the land was in the shape of a square!

Adding a Circle: The Biggest Surprise

In the next cluster meeting, a teacher asked:

"If the square gives maximum area among rectangles, what if we use a rope with the same length as the perimeter (44 m) to make a circle?"

This became the next question:

If a circle has a circumference (perimeter) of 44 metres, what is its radius and area?

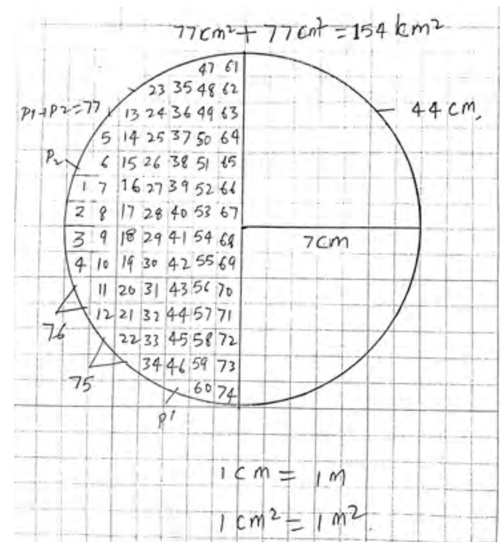


Figure 2

The estimation shown in Figure 2 was made using a graph paper (replacing m with cm). Using the formula, we arrived at:

$$\mathbf{C = 2\pi r}$$

$$\mathbf{44 = 2 \times \pi \times r}$$

$$\mathbf{r = \frac{44}{2 \times 3.14} \approx 7 \text{ metres}}$$

Using the value of r to calculate the area:

$$\mathbf{A = \pi r^2 = 3.14 \times \left(\frac{44}{2 \times 3.14}\right)^2 \approx 3.14 \times 49 \approx 153.86\ m^2}$$

So, the area of the circle is approximately 154 m², which is larger than that of the square (121 m²).

This was an eye-opener. The circle, with the same perimeter as the rectangles, gave the **largest area.**

Teachers Reflect: Is the circle the most efficient shape?

We now had a new insight to explore. Among all shapes that can be formed with the same boundary, the circle encloses the largest area. Teachers reflected on this:

- "Is this why tanks, plates, and pots are often round – because they hold more with less material when the height is the same in 3D shapes?"
- "Does Nature use this property of circles – look at nests, fruits, planets? Maybe it's because it's more efficient."

Table 1. Summary

Shape	Dimensions (m)	Area (m²)
Rectangle	21 × 1	21
Rectangle	20 × 2	40
Rectangle	15 × 7	105
Rectangle	14 × 8	112
Rectangle	11 × 11 (square)	121
Circle (r = 7m)	C = 44	**154**

Here are visual diagrams of different shapes—all with the same **perimeter of 44 metres**—that were discussed in the article. These diagrams can be used in teacher training sessions or classroom demonstrations:

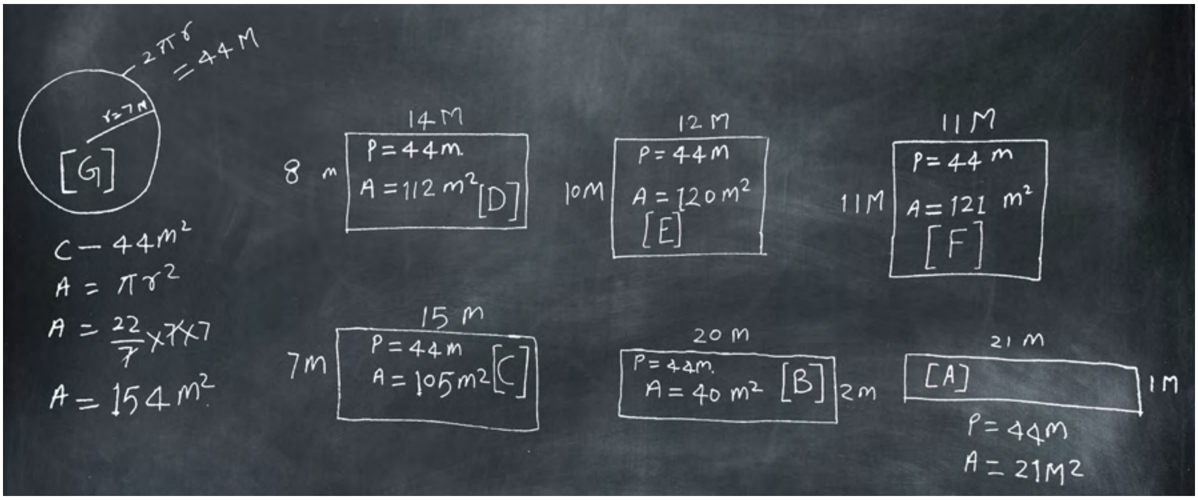


Figure 3

This led to beautiful discussions about protecting agricultural areas, fencing in the garden, making the house, and real-life applications in design and architecture.

Taking the Idea to the Classroom

Inspired by the discussion, we designed a classroom activity for Grade 5 students. We gave students ropes of 44 cm (using thread or string) and asked them to make different rectangular shapes using graph paper.

Children were excited — it felt like solving a puzzle. The results were similar to what the teachers had come up with.

One group made an 11 by 11 square. Another made 14 by 8 rectangle. Some tried extreme shapes such as 20 by 2 or 21 by 1.

They calculated the area for each. To their surprise, the square had the biggest area, even though all shapes had the same perimeter. One child said:

"Sir, jab chaaron taraf barabar ho to zameen zyada milti hai!"
(Sir, when all sides are equal, we get more land!)

That one sentence captured a mathematical truth.

In another class, a boy said, "Sir, agar perimeter fix hai to sabse zyada area gol shape deta hai!"
(If the perimeter is fixed, the round shape gives the maximum area!)

From Rote to Reasoning: Shifting Teaching Practices

This activity challenged the traditional way area and perimeter are taught. Usually, children memorize formulas:

- Area = length × breadth
- Perimeter = 2 × (length + breadth)

By working with a **fixed perimeter** and **changing area**, including the circle, students were forced to **think, test**, and **observe patterns**.

Teachers noted that students who struggled with formula-based teaching were actively participating when allowed to reason and **explore using materials**.

How to Integrate this in Regular Teaching

This concept can be integrated into Grade 4 Math Magic Chapter 13, **Field and Fences**, and Grade 5 Math Magic Chapter 3, **How Many Squares**, and Chapter 11, **Area and its Boundary** in practical and creative ways.

1. **Story context:** "A farmer has 44 metres of fencing material. What shapes can he make for his field to get maximum land?"
2. **Use materials:** Rope, string, paper strips, matchsticks.
3. **Draw and measure:** Let students calculate and compare.
4. **Discuss:** Ask open-ended questions like:
   - "What changes when the rectangle changes?"
   - "What stays the same?"
   - "Which shape gives the biggest area?"
5. **Extension:** Include the **circle** in the conversation. Draw it with a string compass in the graph paper.

Conclusion

Mathematics is not just about speed and accuracy — it's about **sense-making.** The experience of exploring the topic of area and perimeter with teachers and students showed us that **when learning is rooted in exploration, guided by curiosity, and connected to real life, deep understanding can emerge.**

The journey from rectangles to squares, and then to the circle, showed a powerful idea:

For a given perimeter, the circle gives the maximum area.

This is not just a mathematical fact — it's a gateway to critical thinking and appreciation of the natural world. The same thing can be done with other shapes as well and students can note and discuss their observations.

KARAN SINGH has 11 years of experience in education at the Azim Premji Foundation, Rudraprayag, supporting teachers and improving classroom practices in government schools. He may be contacted at karan.singh@azimpremjifoundation.org

"How Do I Know That They Got It?"

Questions that assess mathematical understanding

Kshama Chakravarthy

Have you ever taught a lesson that seemed successful, only to discover later that your students didn't grasp the material as well as you thought? Ever conducted a class that you thought went great, but at the end of the class a student asked you the most basic doubt that made you wonder if they understood anything at all in that session? We've all been there - and this happens due to a variety of factors, such as gaps in communication, differing learning styles, or even moments of distraction that can leave students struggling to grasp key concepts, despite our best efforts in the classroom.

When we introduce a concept, the examples (and non-examples) that we use play a crucial role in the understanding of the concept for a student. For example, showing a triangle always as an equilateral triangle with an upright orientation may lead to the over-generalisation among students that a triangle needs to look that way and that any other orientation or size does not qualify it to be a triangle (Figure 1). Using different examples for triangles, and providing some non- examples too (curved lines, open figures) will help build a proper understanding of triangles.

Similarly, while writing the expanded form of a number, we always tend to split it from left to right the way the digits appear, i.e., 3409 is 3 thousands + 4 hundreds + 0 tens + 9 units. A lot of students don't pay attention to the place name or place value but only notice the order of the digits- 3, 4, 0 and 9. And when you change the order in which the places are called out, they make an error. For instance, 3 tens + 2 units + 8 hundreds will most likely be written as 328, instead of as 832 by these students. Changing the order for the expanded form of a number while dealing with the topic can help in such cases. Of course, the teacher should use his/ her discretion to decide at what stage in the learning they'd like to bring in these additional points.

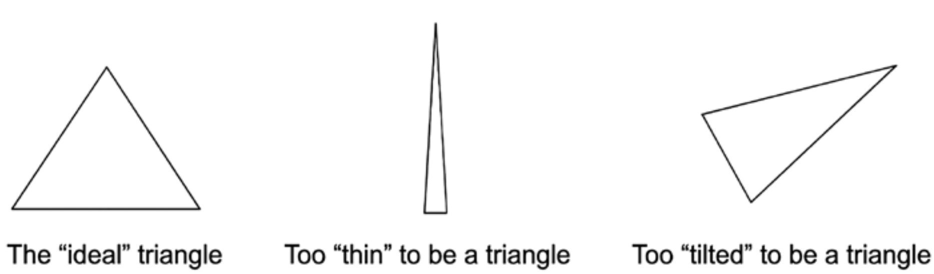


Figure 1

Keywords: Lesson plan, student understanding, assessment, remediation, questions, teaching methods

Consider a few more examples

When we deal with subtraction of whole numbers, we say that we cannot subtract a bigger number from a smaller number, and yet when we move to integers, we teach them how to do exactly that! Also, we say that a zero has no value and yet 10 and 100 are different numbers and so are 1.02 and 1.2. (A lot of students when comparing decimals will say 1.02 and 1.2 are the same or equal because "zero has no value".)

While in the examples of the triangle and expanded form, misconceptions arise because we have not covered all cases, in the rest of the examples, what we say and want them to believe no longer holds true when they move to other topics or advanced lessons. This creates a cognitive conflict in their mind which, for some, takes a while to go away. Being aware of this and knowing how a student is likely to think can help us intervene when required, or prevent the alternative conceptions from forming in the first place.

Once we have incorporated the relevant points in our teaching, a good way to assess if our students have understood a concept in its entirety is to frame the right questions that test their understanding. Well-crafted multiple-choice questions (MCQs) with plausible distractors help identify student misconceptions by presenting incorrect options that reflect common misunderstandings. When a student selects one of these distractors, it provides insight into the specific area or concept they are struggling with (rather than a random guess), enabling targeted feedback and instruction. This is useful because it allows teachers to:

- **Diagnose misconceptions** instead of just checking recall.
- **Differentiate between partial understanding and complete misunderstanding.**
- **Target instruction more effectively,** since the wrong answers reveal patterns in student thinking.

Given below is an example of an MCQ, with the item stem, and the options, also known as distractors, labelled.

Add and choose the right answer:

$$\begin{array}{r} 374 \\ +826 \\ \hline \\ \hline \end{array}$$

(a) 11910
(b) 1200*
(c) 1191
(d) 1190

The item/ question stem: This conveys the question to the student.

Options/ distractors:
The student marks one of these as their answer. There is a rationale behind each distractor added here. Can you figure it out?

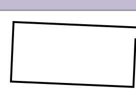
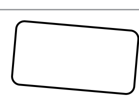
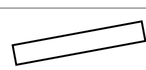
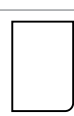
*: Right answer

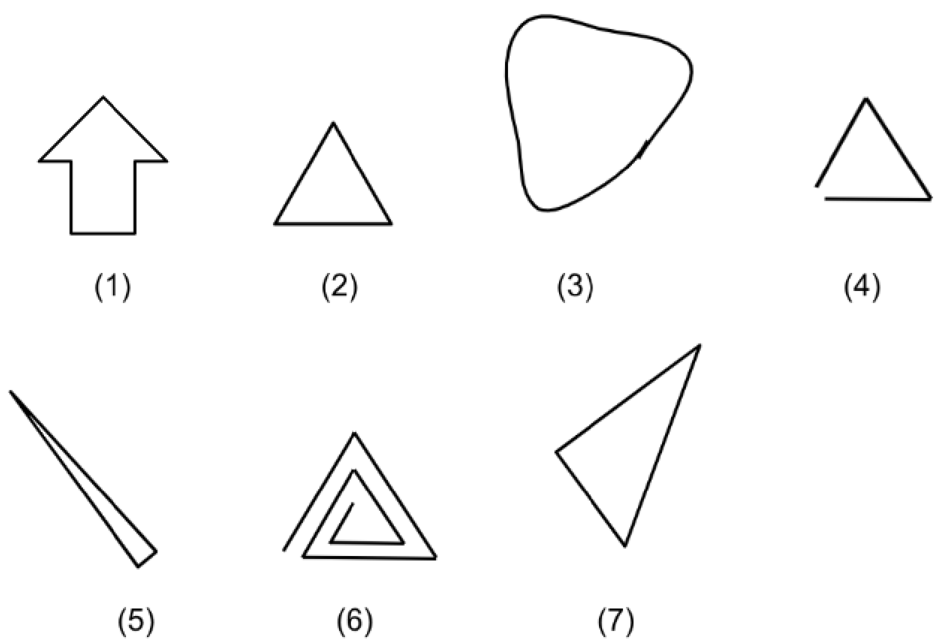
Here are a few sample questions that you can try in class and see if any of the expected misconceptions emerge. Questions like these have been designed to test the understanding of students across the world, irrespective of their socio-economic background, gender, teacher's experience or expertise. The likely logic for choosing each option is provided, so that it helps you as a teacher plan your remediation to address it. They have been mapped to learning outcomes mentioned in the NCERT documents *(Learning Outcomes at the Elementary Stage, NCERT, 2017* for Grade 3 LOs and *Learning Outcomes at the Foundational Stage, NCERT, 2025* for Grade 2 LOs), and give an indication of what outcome can be checked for (either directly or eventually leading to) through the questions asked.

Multiple Choice Question 1		
Topic: Addition of 2-digit numbers with regrouping	**Grade:** 3	**Learning Outcome:** Solves simple daily life problems using addition and subtraction of three-digit numbers with and without regrouping, sums not exceeding 999.

Testing objective	To check the complete understanding of the addition algorithm. In particular to find a missing digit, necessitating the use of subtraction.	
Question	Find the missing digit in this addition problem. 7 6 + ☐ 9 1 2 5	
	Distractors	**Probable reason for choosing the distractor**
Distractor 1	20	Added 6 and 9 in the units place to get 15, and then added the 1 ten carried from 15, to 7 and 12 in the answer, to get 20.
Distractor 2	19	Added 7 and 12 'seen' in the tens place to get 19.
Distractor 3	5	Solved for 7 + ____ = 12 in isolation to arrive at 5.
Distractor 4	4	The right answer. (76 + 49 = 125)

Multiple Choice Question 2		
Topic: Identify 2D shapes (rectangle in particular)	**Grade:** 2, 3	**Learning Outcome:** • Identifies 2D shapes by their names (for example, square, rectangle, triangle and circle) and describes their observable characteristics (for example, the pages of a book are rectangular and have 4 sides, 4 corners) (Grade 2) • Describes 2D shapes by the number of sides, corners and diagonals (Grade 3)

Testing objective	Identify a rectangle	
Question	Select all the rectangles.	
	Distractors	**Probable reason for choosing the distractor**
Distractor 1		"Looks" like a rectangle. Fails to see that the image is open, or may not know that rectangles are closed figures.
Distractor 2		Fails to see the curves or does not realise that a rectangle has 4 corners and is made of 4 straight lines.
Distractor 3		The right answer. (A reason for not choosing this may be that it is too "thin" to be a rectangle.)
Distractor 4		Believes the orientation and size is right for it to be a rectangle. Misses or ignores the curve.
Notes	Students are used to identifying objects that "look" like a rectangle, and sometimes miss the properties that make it (in the mathematical sense) a rectangle. We may have also, without realising, shown rectangles to appear a certain way, which makes them believe that a change in the orientation or a "thin" looking rectangle is not a rectangle at all.	

<table>
<tr><th colspan="4">Multiple Choice Question 3</th></tr>
<tr><td colspan="2">Topic: Identify 2D shapes (triangle in particular)</td><td>Grade: 2, 3</td><td>Learning Outcome:
• Identifies 2D shapes by their names (for example, square, rectangle, triangle and circle) and describes their observable characteristics (for example, the pages of a book are rectangular and have 4 sides, 4 corners) (Grade 2)
• Describes 2D shapes by the number of sides, corners and diagonals (Grade 3)</td></tr>
<tr><td>Testing objective</td><td colspan="3">Identify a triangle</td></tr>
<tr><td>Question</td><td colspan="3">Which of these are triangles?
</td></tr>
<tr><th></th><th colspan="2">Distractors</th><th>Probable reason for choosing the distractor</th></tr>
<tr><td>Distractor 1</td><td colspan="2">(2), (4), (6)</td><td>They “look” like an equilateral triangle in an upright position.</td></tr>
<tr><td>Distractor 2</td><td colspan="2">(2), (5), (7)</td><td>The right answer. [A reason for not choosing this may be that (5) is too “thin” to be a triangle and/or (7) is not in the right orientation.]</td></tr>
<tr><td>Distractor 3</td><td colspan="2">(2), (3), (4), (7)</td><td>They have 3 “sides”. (The fact that a triangle has 3 straight sides or is a closed figure is missed.)</td></tr>
<tr><td>Distractor 4</td><td colspan="2">(1), (2), (4), (6)</td><td>They ‘look” like triangles, even when the number of sides is more than 3, or it is an open figure.</td></tr>
</table>

Multiple Choice Question 4		
Topic: Standard expansion of a 2-digit number	**Grade:** 2, 3	**Learning Outcome:** Applies the understanding of place value of numbers while grouping & recognising them.

Testing objective	Check the understanding of expansion of 2-digit numbers	
Question	3 Ones + 5 Tens = ______	
	Distractors	**Probable reason for choosing the distractor**
Distractor 1	35	Writing the digits in the order in which they appear.
Distractor 2	53	The right answer.
Distractor 3	350	3 ones is 3 and 5 tens is 50. Writing them in that order gives 350 (three and fifty).
Distractor 4	503	3 ones is 3 and 5 tens is 50. Writing the tens first, you get 503 (fifty and three).
Notes	We always tend to write the expanded form starting from the highest place (say, hundreds, then tens and then units) while writing from left to right. Some students build a strategy to write the digits in that order from left to right to form the number, without observing or understanding what the place value is.	

Multiple Choice Question 5		
Topic: Subtraction with regrouping	**Grade:** 2	**Learning Outcome:** Solves daily life situations based on subtraction of two digit numbers.

Testing objective	To check if the student is able to subtract with regrouping	
Question	Solve. 83 – 67 ____	
	Distractors	**Probable reason for choosing the distractor**
Distractor 1	26	Subtract 7 from 13 to get 6, and without accounting for the “borrowed” ten, subtract 6 from 8 to get 2.
Distractor 2	24	7 – 3 in the units place and 8 – 6 in the tens place.
Distractor 3	20	0 in the units place and 8 – 6 = 2 in the tens place.
Distractor 4	16	The right answer.
Notes	Students choosing Distractor 2 are looking at two separate calculations with the order of the digits not mattering (“Always subtract the smaller number from the larger number”). Those choosing Distractor 3 may think that a larger number cannot be subtracted from a smaller number in the units place and so add a 0 there and then proceed to subtract 6 from 8 in the tens place to get 2.	

Multiple Choice Question 6		
Topic: Expanded form of a 3-digit number	**Grade:** 2	**Learning Outcome:** Reads and writes numbers up to 999 using place value.
Testing objective	To check if the student is able to identify the expanded form of a 3-digit number	
Question	What is the expanded form of 461?	
	Distractors	**Probable reason for choosing the distractor**
Distractor 1	4 + 60 + 1	Focus on the order of the digits- 4, 6 and then 1, without looking at place value OR Reading the number as "four sixty-one".
Distractor 2	40 + 6 + 1	461 is seen as forty six and one.
Distractor 3	60 + 1 + 400	The right answer.
Distractor 4	4 + 1 + 6	461 is 4, 6 and 1.
Notes	This is a simple question that checks the understanding of the standard expansion of a 3-digit number. There is an implicit understanding of place value, face value and place name.	

By asking the right questions, teachers can move beyond mere rote memorization and instead foster a deeper understanding of mathematical concepts. By incorporating a combination of both non- MCQs (open-ended, probing questions), as well as MCQs with careful thought out distractors, we can gain valuable insights into our students' thought processes, identify areas of misconception, and adjust our instruction to meet the diverse needs of our students. Here we have explored the MCQs specifically. Ultimately, these approaches enable teachers to create a more supportive and effective learning environment, where students can develop a strong foundation in mathematics and build confidence in their problem-solving abilities.

Teachers interested in trying any of these questions in their class can fill this short form by clicking or scanning the given QR code so that we can brief you on the next steps on how you can incorporate this in class. Tips for possible remediations will also be provided.

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Fact Families

Swati Sircar

A fact family is essentially a group of three natural numbers such that the sum of two of these numbers is the third. For example, {2, 3, 5} forms a fact family since 2 + 3 = 5. But {7, 4, 2} does not form a fact family. In a fact family, the order of the elements does not matter. If two of the numbers of a fact family are identical, the third number will either be 0 or it will be twice the repeating number, i.e., if 5 and 5 are the repeating numbers then {5,0,5} and {5,5,10} are both fact families. Is it possible for all three members of a fact family to be identical? Yes - provided the members of that fact family are each equal to 0!

This article is based on my observation of and reflection on two consecutive classes taught by Aakefa Basri in Class 2, Azim Premji School, Yadgir. She obtained the key idea from https://bit.ly/4qlk2FF

In the first class, the teacher initiated the topic with a discussion on the notion of a family – father, mother, child and then introduced the family of three numbers 4, 5 and 9. She asked if they could see how these numbers are related using addition and subtraction. The addition facts generated were 4 + 5 = 9 and 5 + 4 = 9. When some students wanted to include 2 + 7 = 9, the teacher reminded them that the members of this family were only 4, 5 and 9. She also demonstrated this with counters as shown in Figure 1. Then she asked for subtraction facts with the members of the same family, and got 9 – 5 = 4 and 9 – 4 = 5.

Now the teacher asked them to make a fact family with families which they themselves created. The initial choice of {6, 3, 10} became {6, 3, 9} after some verification with blocks. When the teacher specified that no members of the family were to be repeated, {4, 4, 8} was changed to {3, 5, 8}. Group work generated the families {12, 8, 20}, {54, 31, 23} and {20, 4, 16}.

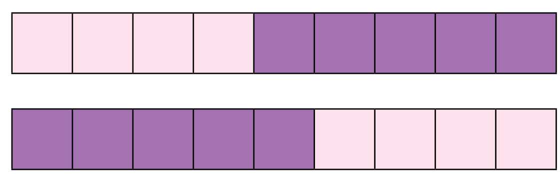

Figure 1

For the second class, Aakefa took them to the playground and asked them to collect as many pebbles as they could in one minute. They then counted the pebbles collected. Each had a different number. The teacher asked them to split each collection into two parts and thus generate a corresponding fact family. One student had 43 pebbles and split them into 42 and 1. Her fact family was {1, 42, 43} and she noted down that 1 + 42 = 43, 42 + 1 = 43, 43 – 1 = 42, 43 – 42 = 1.

When one student got the family {20, 30, 50}, the discussion then went on to other fact families of which 50 was the largest member, such as {10, 40, 50} and {50, 0, 50}. They saw that the collection of 50 pebbles could have been split in many ways. They started

Keywords: Numbers, addition, subtraction, observation, manipulation

exploring subtraction facts for 50 instead of addition facts.

Some children went further and came up with fact families involving 50 but not as the highest number, e.g., 60 – 10 = 50. A classic case of how math starts from concrete but becomes more abstract. Students were able to make fact families with single digit numbers as well as double digit ones. However, it was interesting to see children avoiding addition or subtraction facts involving regrouping, such as 37 + 25 = 62 or 51 – 24 = 27. They clearly preferred facts involving only digit-by-digit addition/subtraction, e.g., 23 + 14 = 37 or 45 – 13 = 32.

So, any fact family, especially with three distinct numbers, generates two addition facts, e.g., 2 + 3 = 5 and 3 + 2 = 5 illustrating the commutative property of addition. Similarly, there are two subtraction facts, e.g., 5 – 2 = 3 and 5 – 3 = 2 illustrating that the addition fact can be expressed as two subtraction facts.

Connecting addition and subtraction facts this way is crucial for deciphering word problems in the long run.

Moreover, it provides the opportunity to discuss that the biggest number is the sum of the remaining two, i.e., a collection of objects representing the biggest number can be split in two portions, each representing one of the remaining numbers. The second day's activity of collecting pebbles was geared towards this.

Observations

1. The pedagogic shift from concrete to abstract is preferable to the reverse shift observed, and children could have been asked to take a handful from a bag of pebbles brought into class and construct the fact families. It is important though, that they pick- rather than being handed – pebbles. There is greater involvement of the learner and each gets a random number of objects – a number that is not preselected by anyone.
2. Since this was done with Class 2, 2-digit numbers were involved. But this activity can and should be initiated with Class 1 after they learn numbers up to 20 and with just single digit addition/subtraction. This can facilitate automatization (see [1]) of single-digit addition and corresponding subtraction facts – both critical for fluency in any addition-subtraction. This can therefore provide a lot of opportunities to sharpen mental math. And such an activity will get them to start playing with numbers which is essential to befriend (and fall in love with) math!
3. Variations that can be explored by teachers:
   - Find all possible Fact Families such that the biggest number is (say) 20. How many fact families are possible with 20 as the biggest number?
   - Find all possible Fact Families such that the smallest number is (say) 3. How many fact families are possible with 3 as the smallest number?
   - Create a fact family such that two of the numbers are from the same multiplication table, e.g., 15 and 35. What can you say about the third number? Can you explain why this happens?
4. Multiplication Fact Families such as {2, 3, 6} can be generated with multiplication-division, and can help students master multiplication facts at random. Further explorations can include:

   A) Find all possible multiplication fact families such that the biggest number is (say) 20. How many fact families are possible with 20 as the biggest number?

   B) What happens if 1 is in a Fact Family? What about 0?

   C) A cross-number puzzle based on addition fact families is provided in the same issue.

A) can pave the way to finding all possible factors which is an important skill with applications such as middle term factorization- later used for solving quadratic equations. B) on the other hand, drives home the uniqueness these two special numbers have with respect to multiplication-division.

Based on these ideas, the following worksheet on Addition Fact Families was created:

Fact-Family Worksheet (Addition-Subtraction)

Fact-Families are a group of 3 numbers {a, b, c} such that $a + b = c$

Class 1

1. Find the addition facts and the subtraction facts for each fact-family:
   a. {13, 9, 4}   b. {6, 9, 15}

2. Which of the following number triplets are fact-families?
   a. {3, 5, 8}   b. {7, 2, 4}   c. {9, 4, 1}   d. {6, 2, 8}   e. {11, 4, 7}
   f. {7, 9, 2}   g. {1, 4, 7}   h. {5, 12, 6}

3. Make your own fact-families as follows:
   a. The sum is 13
   b. The smallest number is 5
   c. The two smaller numbers are less than 7
   d. The biggest number is more than 8

4. How many fact-families can you make such that
   a. The biggest number is 7
   b. The biggest number is 10

Class 2

1. Make fact-families such that:
   a. The biggest number is less than 50 and the smallest number is greater than 20
   b. The biggest number is greater than 40 and the smallest number is less than 15

2. How many fact-families are possible such that the biggest number is
   a. 7   b. 8   c. 10   d. 13   e. 19   f. 22
   g. Can you guess how many fact families will have 37 as the biggest number?

3. How many fact-families are possible with the smallest number as 6?
   Give two examples.

4. How many fact-families are possible such that the middle number (neither smallest, nor biggest) is
   a. 5   b. 8   c. 11   d. Guess how many if it is 73?

5. Can 0 be in a fact-family? Give an example.

Reference

1. Addition pullout: https://bit.ly/4o5Q5YC
2. Subtraction pullout: https://bit.ly/48BAHON
3. Commutative property of addition: https://bit.ly/4nXcTcZ
4. Word problems: https://bit.ly/4odUfxq
5. Word problem Worksheet: https://bit.ly/49lXhuW

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Interpretation of the 'Art in Numerals' in page no 8 of the July 2025 issue

18	21	24	27	30	33
23	26	29	32	35	38
28	31	34	37	40	43
33	36	39	42	45	48
38	41	44	47	50	53
43	46	49	52	55	58
48	51	54	57	60	63

Similar to the given pattern shown in page 19, we can create an $m \times n$ grid with another rule.

Readers, can you find a general representation for any number in this grid? Will there be a relationship between numbers situated symmetrically around a given number?

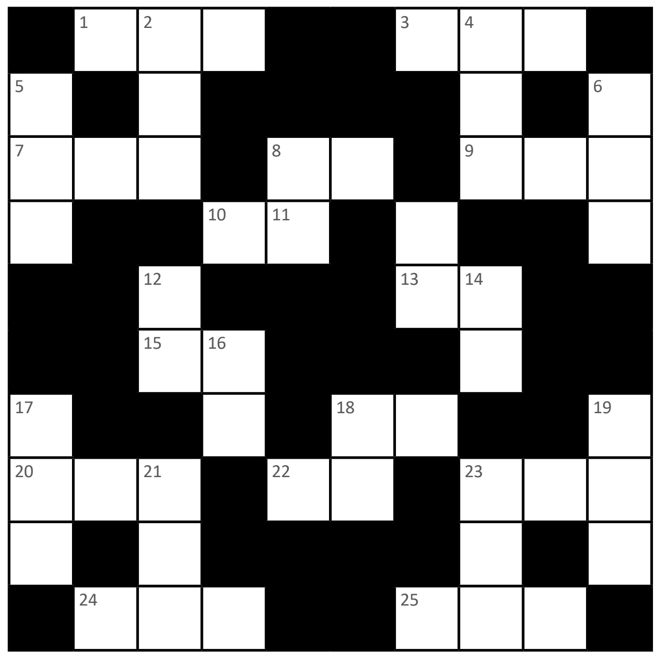
Dr. J. Sekhar, School Assistant (Maths), ZPHS Chavarambakam, Andhra Pradesh

Cross-Number Puzzle

Swati Sircar

This cross-number puzzle is a follow up to the article on Fact Families in the same issue. An addition fact family is essentially a set of three natural numbers, such that the sum of two of these numbers is the third. For example, {2, 3, 5} forms a fact family since 2 + 3 = 5. But the set {6, 4, 1} does not form an addition fact family.

Note: Since a fact family is a set, the order of the elements does not matter.



Solve the cross-number puzzle

Across

1: The biggest possible number in the addition fact family with 307 and 50

3: This number forms an addition fact family with 200 and 14

7: The smallest number in the addition fact family with 250 and 126

8: The biggest possible number in the fact-family with 14 and 42

9: This number forms a fact-family with 170 and 600

10: The smallest member of the fact-family with 36 and 68

13: This number forms a fact family with 25 and 100

15: This number forms fact families with {7, 11} as well as {3, 15}

18: This number forms fact families with (37, 3} as well as {40, 6}

20: The biggest possible number in the fact-family with 300 and 202

22: This number forms a fact-family with 39 and 100

23: The smallest number in the fact-family with 700 and 396

24: This number forms fact-families with 500 and 40 as well as with 300 and 160

25: This number forms fact-families with 300 and 200 as well as with 900 and 400

Keywords: fact families, addition, subtraction, puzzle, crossword

Down

2: This number has digits which form a fact-family in which each of the members is less than 6.

4: The biggest possible number in the fact family with 520 and 304

5: This number forms a fact-family with 209 and 210

6: This number forms a fact family with 331 and 471

8: This number forms a fact-family with 60 and 8

11: This number forms a fact-family with 70 and 23

12: This number forms a fact family with 100 and 29

14: This number forms a fact-family with 27 and 26

16: This number forms a fact-family with 101 and 17

17: The biggest possible number in the fact-family with 425 and 525

18: This number forms a fact-family with 15 and 16

19: The biggest number in the fact-family with 687 and 153

21: This number has digits which form a fact-family in which each of the members is less than 7

23: This number forms a fact family with 444 and the number in 7 Across.

Solution to the Cross-Number puzzle found on Page 52

Think of a four-digit number 'abcd' and move the first digit 'a' to the end so that it becomes the last digit of the number which would be 'bcda'. Subtract the new number from the old.

For example, if the first number 'abcd' is 3568, then the next number 'bcda' is 5683.

In the March 2025 issue, we noticed a relationship that helped us guess the original number if we were given some data related to these two numbers. In this issue, we make the same claim. If I know the difference between the two numbers and the first digit of the original number, then I can guess that number. [In the given example, if I know that the difference is -2115 and the first digit was 3, I can guess that the first number was 3568]

Can you guess how I do this? Try with some other 4-digit numbers. Now try to generalise this trick for an n-digit number! Send in your solutions to AtRightAngles.editor@apu.edu.in

Contributed by Yathiraj Sharma

Investigating Perimeter and Area with Square Tiles

Mohan R

Understanding the relationship between perimeter and area can be an exciting exploration for students in grades 3 to 7. Square tiles offer an ideal hands-on method to discover and visualise these concepts intuitively. In this article, we share a set of practical activities involving square tiles. These activities help students explore the relationship between shape, area and perimeter, and learn to optimise shapes to minimise perimeter or maximise area.

Materials required

- 5 cm × 5 cm square tiles cut from a thick chart paper
- Pencil and paper for notes
- Grid paper/graph paper for drawing and rough work.

Activity 1: Starting Small – Classes 3-5

In the new NCERT textbooks, the notion of measuring area by counting unit squares is introduced in Class 3, and measuring length is introduced in Class 4.

The teacher could begin with a short introduction/revision of the notions of area and perimeter, and how to measure them using square tiles. (See Figure 1.) The class is divided into groups of two, and each student is asked to create rectangles with exactly 12 tiles (orientation does not matter). The pairs discuss and find different ways to arrange these tiles and then carefully record each arrangement on grid paper. Students are then asked to count and record the perimeter (which changes), and the area (which is always 12 square units).

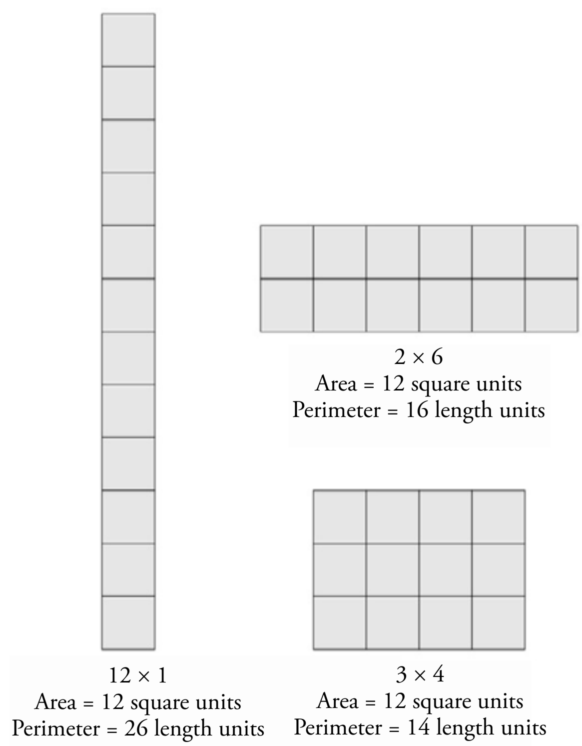


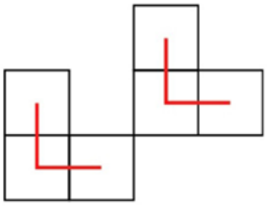
Figure 1: Rectangles formed with 12 square tiles

Keywords: Area, Perimeter, Square tiles, Polyominoes, Floor plans

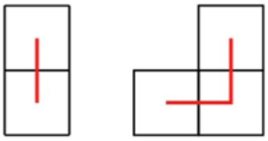
Figure 2: An illustration of children playing with tiles. (Created with GPT)

Activity 2: Fencing the floor – Classes 4-6

Once students are comfortable creating rectangles using a given (small) number of square tiles, the teacher can extend the investigation by inviting students to create shapes that aren't necessarily rectangles. For clarity, we can refer to these new shapes as "floor plans." The only requirement for these floor plans is that they must be **connected**, which means it is possible to start at any tile and reach every other tile by moving only across shared edges (Figures 3 and 4).



(a) An example of a floor plan that is not connected - the two L tiles are not connected via a common edge



(b) An example of a floor plan that is not connected - there are two disconnected components

Figure 3. Two floor plans that are not connected - Travelling along the red line we cannot cross along the edges.

The objective of this activity is to identify the floor plan with the smallest possible perimeter. In practical terms, this means finding the arrangement of tiles that would minimize the amount (and therefore the cost) of fencing needed around the boundary. To start, the teacher provides each student with exactly six square tiles. Students are then tasked with creating and recording all possible connected floor plans on grid paper, noting that orientation does not matter (rotations or reflections are considered the same) (Figure 5). After drawing all possible floor plans, students calculate and record the perimeter of each shape.

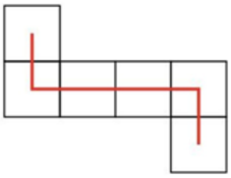


Figure 4. An example of a connected floor plan. Travelling along the red line we can reach all the tiles by crossing edges.

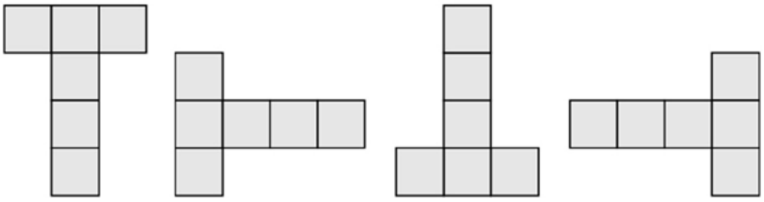


Figure 5. Rotations or reflections of a floor plan do not change the perimeter or area.

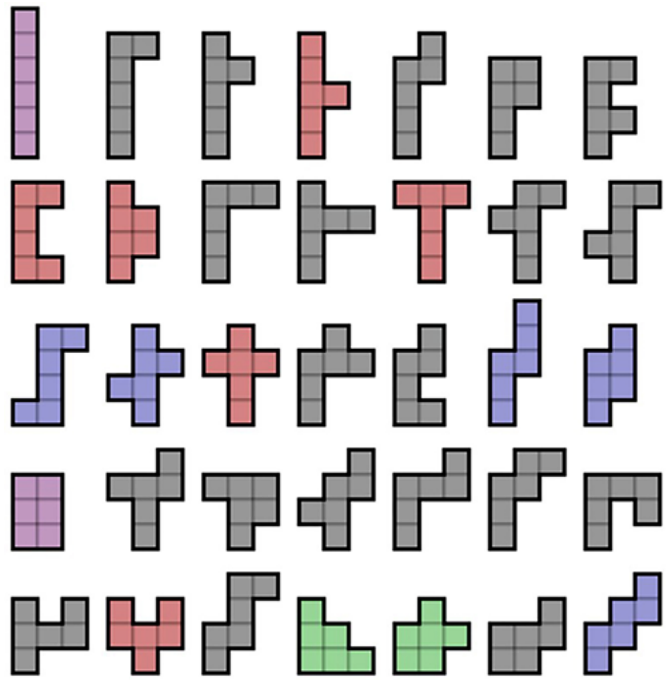
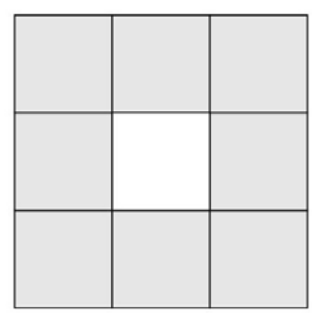


Figure 6. All floor plans that can be created using 6 square tiles. Credits: By R. A. Nonenmacher - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=4773113

Note: It is possible to have a floor plan with a blank interior as in Figure 7. In such cases, the perimeter is the sum of the interior boundary and the exterior boundary.

The teacher discusses the following questions, and the students *reason and argue* why their answers are correct.

1. Which arrangement had the greatest perimeter? Which arrangement had the smallest?
2. Why do you think changing the arrangement affects the perimeter but not the area?
3. Do you see any relationship between the rectangles and the factors of 12?
4. What do you think happens if we take 13 tiles instead of 12?
5. What do you think happens if we take 24 tiles instead of 12?
6. Will the perimeter always be even?
7. (For advanced learners) What do you think happens if we take n tiles instead of 12?



Area = 8 units
Perimeter = Length of outer boundary and inner boundary.
So Perimeter = 12 + 4 = 16 units

Figure 7. A floor plan with 8 square tiles

Activity 3: Conquer the land – Classes 5-7

The final activity in this series explores a related but converse question: Given a fixed perimeter, what rectangle maximizes the enclosed area? This problem could be presented as a real-world scenario: Imagine conquering land represented by square tiles, and you have only enough fencing material of a specific length (the perimeter). How can you arrange your boundary so that the land you've conquered covers the greatest possible area?

To begin, the teacher sets the perimeter at 24 units. Students then generate all possible rectangles with this fixed perimeter, carefully calculating the area for each configuration to identify the one with the largest area. At this stage, teachers can gently introduce the language of algebra, tables and systematic reasoning to help students organise their findings and guide their thinking. After practising with several small perimeter values, students who are comfortable with rectangles can then be challenged further: explore non-rectangular floor plans to find which shapes offer the greatest area given a fixed perimeter.

Conclusion

Through these interactive and exploratory activities, students intuitively grasp key mathematical ideas related to area and perimeter. They discover important patterns, engage in systematic reasoning, and experience how mathematical concepts connect to practical scenarios. Using simple square tiles, these activities build foundational skills and a deeper appreciation for mathematics as being both creative and practical.

For more ideas on this topic

1. Some previous articles of At Right Angles have discussed similar ideas. For example the tearout https://bit.ly/3L0uH8u - this includes similar explorations on a square grid.
2. One can start with smaller floor plans with $n = 3$ and find pairs with a) same area and same perimeter b) same area, different perimeter c) same perimeter, different area.
3. There is a notion of L-ing (cutting an L from the corner of a rectangle) preserving perimeter but decreasing area and U-ing (cutting a U from a side of a rectangle) reducing area while increasing perimeter discussed in the link above. This can be explored with polyominoes. See tasks 4 and 5 in https://bit.ly/47i5jCt
4. There is another Tearout on hexominoes https://bit.ly/49fKkmu

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Children of Plato: A Mathematical Roleplay

Padmapriya Shirali

Over the years, I have found ways to convert maths-based stories into short skits that have proven enjoyable for the students. This activity makes mathematics more accessible and exciting, both for the actors and the audience.

Mathematical plays can support learners with diverse abilities as well as promote collaboration and creativity. It holds the potential to create a positive attitude, to remove the monotony of pen-and-paper problem-solving, and to foster a shift in the student's perception of the subject. It might also alleviate math anxiety.

This script is part of a larger script that I had worked on while adapting the storybook The Phantom Tollbooth by Norton Juster. In this story, the main character travels to different imaginary lands. The following scene is his visit to a land called Digitopolis. I have changed the characters and introduced new elements into the story.

Five awesome friends for Chinnu!

Chinnu is walking around in the land of shapes and meets a dodecahedron. This weird shape has many faces and a different expression on each face.

Wow! You have so many faces. Who are you?

Chinnu

(Turning round and round) I'm a dodecahedron—a shape with twelve faces *(begins to turn around again)*

Dodecahedron

Keywords: Platonic solids, pedagogy, inclusive learning, role-play, team-work.

Hey! I am getting dizzy seeing you spin round and round. Stop, please!

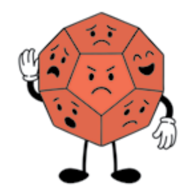
(To the audience) That's a lot of faces!

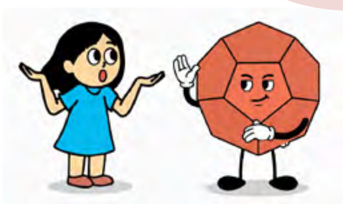
Why do you need twelve faces?!

You mean where you come from, fellas like you have only one face?

Chchch... *(pitying Chinnu)* You will wear out your face using it for different expressions.

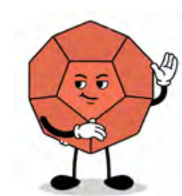
Look at me, I have one for smiling, one for grinning, one for crying, one for frowning ....





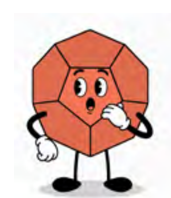
And each of your faces has 5 edges!
(feeling him around the face)

(haughtily) Each one is shaped like a perfect pentagon.



Oh. This is a pentagon. I always wondered what it was..

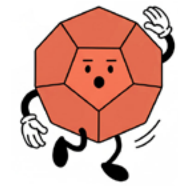
(correcting Chinnu) It is a regular pentagon; all my sides and corners are equal.



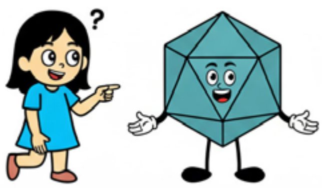
Hmm. *(Observing the equality)* Can you roll?
(Nudges the dodecahedron)

Only if someone rolls me too fast! But mostly, I like to sit still and show off all my faces.
(turns grinning face to audience)

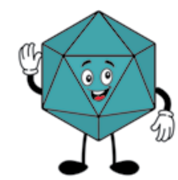
Is the rest of your family also like you?



In some ways, yes and in some ways, no.
Here comes my younger brother, the Icosahedron
(icosahedron walks in)



Oh My! You are equally interesting and you have even more faces *(trying to count)*



Yes. I'm the most "faceted" of us all, with 20 triangle faces.

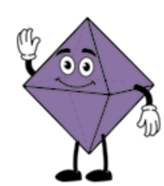
Icosahedron

I wonder which of you two can roll best!

That would be me! With my 20 faces, I roll like a super-fancy dice.

I roll too, but not as far or as wild as Icosahedron!

Enter Octahedron

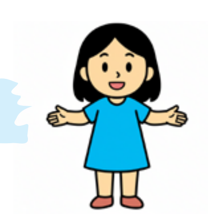


Hey, fellas! What's happening here? Who is this weird one-faced fellow?

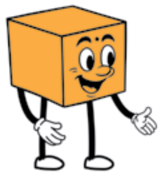
Octahedron

Tetrahedron and Cube also enter, and they all stare curiously (turning their sympathetic faces to Chinnu). Chinnu shrinks a little and begins to feel conscious of her single face.

This is my other brother, an octahedron and here are my two sisters.



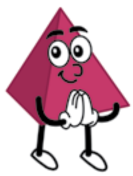
(to the audience) What difficult names!



Cube

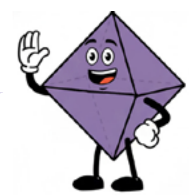
(Shaking hands) Hi! I'm a Cube—the boxy one. I have 6 square faces.

(Talking to herself) Looks like the fancy blocks I played with!



Tetrahedron

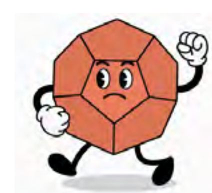
(Does Namaste) I'm a Tetrahedron. I have 4 faces and they're all triangles. But, I'm pointy and a bit mysterious! *(stressing)*



Don't forget me! I'm an Octahedron. I have 8 triangular faces *(spinning around)*

(to the audience) Looks a bit like two pyramids stuck together.

(to dodecahedron) Are there more in your family?



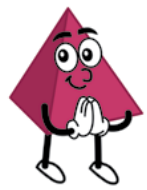
No. We belong to the special Platonic solids family. We do have relatives who are also interesting in their own way.

I am so glad to have met all of you.

We are glad to meet you. It never occurred to us that one face could show so many emotions!

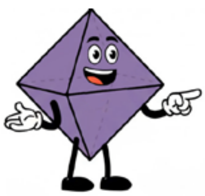
Chinnu: Just one thing I don't understand. Why are you called "platonic solids"?

Ha! We're called "platonic" because a smart person named Plato studied us a long time ago. He thought we were special shapes that make up the world.



We have a lot in common. All our faces are the same shape and size, and our corners and edges are all equal.

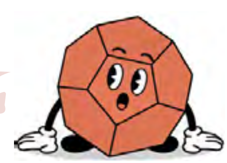
What are you made of?



We can be made of anything—wood, plastic, even jelly—if you make our faces all the same!

Just take care not to eat your math homework after making us! *(all laugh)*

If I built you with paper, would you be strong?

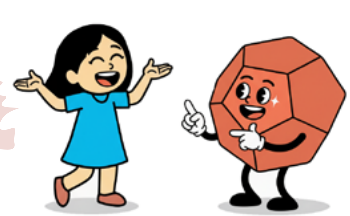


If you fold me carefully, I'll be surprisingly tough! I love being colourful, too.

Can I colour each face differently?

I'd love that! But you will need more colours than there are in the rainbow for that! Let's see how creative you are

(starts work with some crayons)
You're the coolest shape I've ever met!



Thank you! I'm happy to be your geometric friend.

Teachers often draw on varied classroom experiences to help students connect with mathematical ideas. Roleplay is one such approach, although it is not common in mathematics classes. This article offers a ready-to-use scene that the readers may adapt for their learners. Use it to build vocabulary, prompt observation, encourage reasoning, and support participation.

Roleplay invites multiple voices, lowers anxiety, and allows students to "speak" mathematical language in context. We invite the reader to reflect on the following questions.

- Which elements of the scene will you keep or change for your class?
- How will you ensure inclusivity and shared participation?
- What changes could strengthen vocabulary, reasoning, or visualisation?
- How might you extend the storyline for your students?

Roleplay brings mathematics to life: students speak the language of shapes, notice structure and properties, and practise precise vocabulary in a joyful setting. This short scene is a springboard to adapt the lines, redistribute parts for broad participation, add props or movement, and connect to follow-ups such as building nets. Above all, let learners co-author the story. When students enact ideas like faces, edges and vertices, understanding becomes shared and deep.

Reference

1. Juster, N. (2005). The phantom tollbooth (Illustrated by J. Feiffer). Yearling. (Original work published 1961).

Padmapriya Shirali is part of the Community Math Centre based in Valley School (Bangalore) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. In the 1990s, she worked closely with the late Shri P K Srinivasan. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box.' She is currently part of the NCERT textbook development group. Padmapriya may be contacted at padmapriya.shirali@gmail.com

Interpretation of the 'Art in Numerals' in page no 8 of the July 2025 issue

18	21	24	27	30	33
23	26	29	32	35	38
28	31	34	37	40	43
33	36	39	42	45	48
38	41	44	47	50	53
43	46	49	52	55	58
48	51	54	57	60	63

Similar to the pattern shown in page 19, we can create a $n \times n$ grid with another rule.

Can you find a general representation for any number in this grid? Will there be a relationship between numbers situated symmetrically around a given number?

Dr. J. Sekhar, School Assistant (Maths), ZPHS Chavarambakam, Andhra Pradesh

Teaching Mathematics Through Problem-Solving: A Pedagogical Approach from Japan

By Akihiko Takahashi
Reviewed by Anusha T

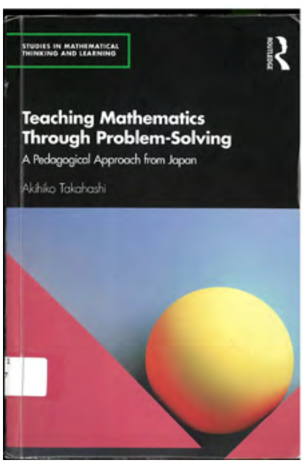


Figure 1

This is a review of the book titled *Teaching Mathematics Through Problem-Solving: A Pedagogical Approach from Japan* written by Akihiko Takahashi. The book was published in the year 2021 by Routledge, Taylor and Francis Group. The author, Akihiko Takahashi, is currently a Professor Emeritus at DePaul University, Chicago, Illinois. He was a mathematics teacher in Japan and later served as an Associate Professor at DePaul University. With over two decades of experience in Lesson Study, he has played a vital role in bridging research and classroom practice. He conducted public research lessons, published widely on problem-solving and reflective journals, and promoted innovative teaching approaches, earning global recognition as a pioneer in Lesson Study research and practice.

The book is a significant addition to mathematics education literature as it brings together decades of Japanese classroom practices and research, and explains them in a way that both teachers and researchers can understand. Adding to its strength, the book gives a detailed explanation of the pedagogical approach called Teaching Through Problem-Solving (TTP). I am deeply impressed by the way the book connects theory with what actually happens in classrooms. Often, educational ideas stay stuck in research papers and do not reach teachers, but this book makes those ideas practical and usable in classrooms. The author bridges this gap by providing real lesson examples, classroom stories, and practical teaching strategies. His writing does not only speak to researchers but also gives teachers and content developers ample concrete ideas that they can try.

For many years, Japanese students have consistently performed very well in mathematics in international tests like TIMSS and PISA[1], but what is often overlooked is how they are taught.

1 Ikeda Y, Kita Y, Takagi R, Suzuki K, Mammarella IC, Caviola S, Lanfranchi S, Pulina F, Giofrè D. The Abbreviated Math Anxiety Scale (AMAS): Applicability and Utility in a Sample of Japanese Elementary School Children. Int J Psychol. 2025 Apr;60(2):e70015. doi: https://doi.org/10.1002/ijop.70015PMID: 39933572; PMCID: PMC11813552.

Keywords: Teaching through problem solving (TTP), Lesson study, Collaborative Lesson Research (CLR), Neriage (Classroom discussion) and Neriage Maps, Mathematics Education, pedagogy, practice, problem solving

In many Indian classrooms, problem-solving is often seen as something to follow up after the main lessons, i.e., once the concepts are taught and the formulae are explained. In Japan, however, the process is reversed: the problem itself is the lesson. Students are introduced to a well-chosen problem, and by working on it, they discover new concepts and methods. This book gives us a clear picture of this teaching method. The book explains how Japanese teachers structure such lessons, what kinds of classroom discussions (called *Neriage*) happen, and how students' thinking develops step by step. This is important for all the stakeholders of mathematics education because it offers an alternative to the lecture-and-practice model that dominates in many countries. It shows us how mathematics can be taught in a way that is both challenging and joyful for students.

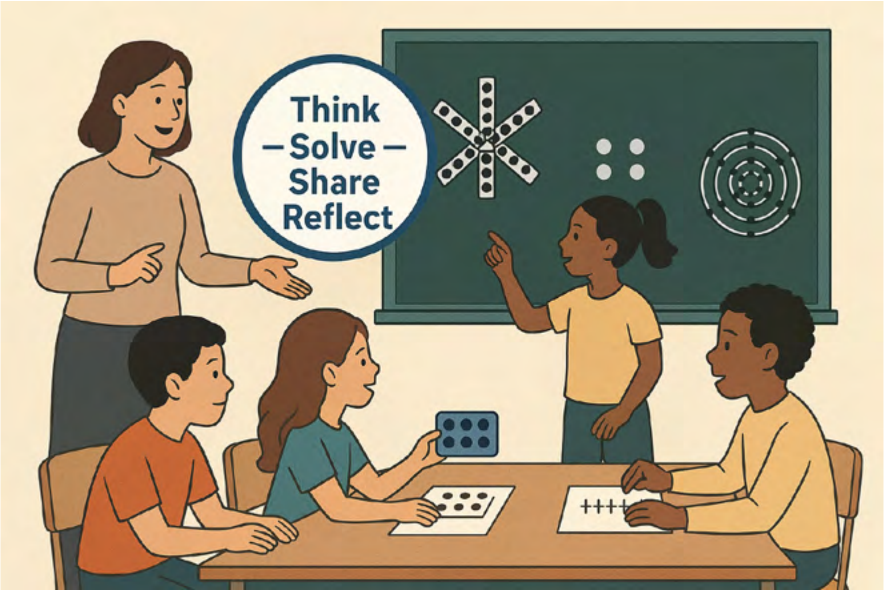


Figure 2 (Generated with ChatGPT)

Finally, the book highlights the role of teacher collaboration. Teaching through problem-solving cannot succeed if teachers work in isolation. The Japanese system of *Jyugyou Kenkyuu* (lesson study) shows how teachers can work together to design, observe, and improve lessons. This makes the book important not just for teaching mathematics, but also for thinking about how teachers can develop professionally.

Overview of the Book

In the **first chapter**, "Development and Major Concepts of Japanese 'Teaching Through Problem-Solving' (TTP)," the author traces how Japanese teachers and schools collaborated to develop and spread TTP as a powerful pedagogical approach. He clearly outlines the three kinds of TTP lessons and the four types of *Neriage*, that structure these lessons. This chapter provides the conceptual foundation for understanding how TTP can be thoughtfully designed and effectively taught.

Chapter 2, "TTP Lessons You Can Use," is particularly engaging as it provides a wide range of classroom-ready TTP lessons, with multiple examples for each lesson type. What makes this chapter stand out is the way it balances theory with practicality that teachers can immediately see how abstract ideas of TTP take shape in real classroom contexts.

Chapter 3, "Designing Your Own TTP Lessons," serves as a practical guide for teachers ready to take the next step. One of the most interesting ideas here is the introduction of *Neriage* Maps, simple yet powerful sketches that help visualize the flow of whole-class discussions. These maps not only make lesson planning more structured but also give teachers a concrete tool for facilitating meaningful mathematical dialogue.

Chapter 4, "How TTP and Collaborative Lesson Research (CLR) Can Change Your School," really stood out to me because it goes beyond just what happens in one classroom; it talks about how an entire school can grow when teachers work together and learn from each other. The author situates TTP within the Japanese practice of Lesson Study, showing how teachers in Japan collaboratively refine their lessons through systematic observation and discussion. What I found particularly valuable is his introduction of Collaborative Lesson Research (CLR), an adaptation designed for teachers outside Japan. By presenting CLR as a practical model, the chapter highlights how schools worldwide can build a culture of collective inquiry, making TTP not just a teaching strategy but a driver of school-wide improvement.

What I Like About the Book

I really like the content discussed in Chapter 2 because it establishes problem solving as an integral part of every lesson for every grade, with a handful of well-chosen examples rather than just theory. What I appreciate the most is that the TTP lessons in the book are not generic—they are drawn from actual Japanese classrooms and show how TTP can be used to introduce new concepts, expand understanding, and promote mathematical thinking through open-ended problems with multiple correct answers. The chapter also gives scope for teachers to adjust the lessons based on students' prior learning and classroom needs, which makes it very usable in diverse Indian contexts. Lessons from across the sections give students the space to struggle, explore, and articulate their thinking, which can lead to much deeper learning.

The Japanese school system includes six years of elementary education (ages 6–12) and three years of lower secondary education (ages 12–15), followed by three years of upper secondary education (ages 15–18). The author presents various examples of TTP lessons across schooling stages. The author divides Chapter 2 into three sections; in section 2.1 he gives five TTP lessons where each unit includes 3-4 progressive lessons. In sections 2.2 and 2.3, the author gives spotlight lessons. The three sections are discussed below with a few examples.

Section 2.1: Lessons to develop conceptual and procedural understanding

In this section, the author discusses units *"Can you add these numbers without counting one by one", "Ideas of quantifying crowdedness and speed", "Deriving the Area Formula of Parallelogram", "Introducing fractions"* and *"Building a bridge from Arithmetic to Algebra".* All the units are interesting, but I would like to discuss the unit *"Building a bridge from Arithmetic to Algebra".* In the lessons the author establishes the crucial foundation of learning algebra. The TTP lessons are well-structured and build progressively, encouraging students to think deeply rather than rely on rote methods. Through problems like organising the dots and the stick arrangement task, students learn to generalize patterns, create and test mathematical expressions, and reason without direct counting. They also explore algebraic thinking through quasi-variable concepts and discuss arithmetic rules like parentheses and order of operations. The focus on reasoning, expression, and verification helps students develop strong foundational skills in mathematical thinking, making this section a valuable resource for teachers.

For example, the TTP lesson *'Let's think about ways to count the number of dots'* discusses different ways of counting dots and representing them using mathematical expressions. This lesson is designed to help students explore the generalizability of mathematical expressions and connect symbolic expressions with concrete representations. Figure 3 illustrates the various strategies students used to count the dots and the mathematical expressions that emerged, as discussed by the author in the book.

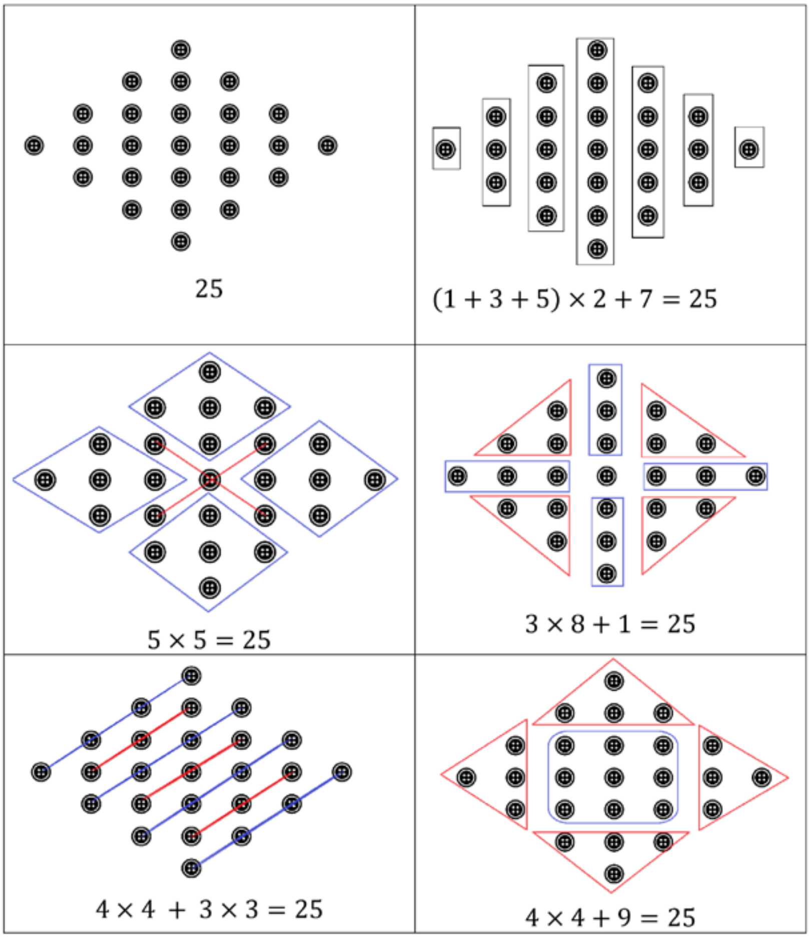


Figure 3: Making groups of buttons and representing ways of counting using mathematical expressions.

Section 2.2: Lessons to expand understanding

In this section, the author gives stand-alone lessons to challenge students' mathematical thinking and ability to solve problems. The author calls these as "spotlight lessons" to highlight that these can be added to the existing curriculum. Spotlight lessons are *"Curious Subtraction", "Comparing areas using pattern blocks", "Let's Make a calendar", "Finding the area of triangles inside parallelogram", "Devising ways to construct a congruent triangle"*. All the lessons are interesting; to understand the nature of the lesson in this section I will discuss *"Let's Make a Calendar"*. In this lesson children are expected to use their understanding of place value and properties of operations to perform multi-digit arithmetic. The pedagogical process undertaken here to find the least number of cards needed to make numbers from 1-31, and the productive *Neriage* that results in development of logical progression in the process are both very impressive. This lesson gives scope to deepen understanding of base ten place value notation as well as geometry.

Section 2.3: Lessons with Problems having multiple correct answers

In this section, the author discusses several spotlight lessons: *"Opening a Cube," "How Many Different Squares Can You Make on a Geoboard?", "Find All the Isosceles Triangles on a Geoboard," and "Let's Create a New Math Problem! (A Lesson from the Book Mondai kara Mondai e)"* All these lessons present open-ended problems with multiple correct solutions, encouraging students to develop higher-order thinking skills (Becker & Shimada, 1997). The lesson "Let's Create a New Math Problem!" is designed for all grade levels and was developed by Japanese researchers and educators. It is taken from the popular Japanese book Mondai kara Mondai e (Takeuchi & Sawada, 1984). This lesson provides an opportunity to extend the "stick problem" discussed in previous sections by creating new problems. For example, a minor modification to the original problem leads to new questions such as: "If we make 50 adjacent squares using sticks of the same length, how many sticks do we need?" or extending to making adjacent cubes. The Neriage map of *Let's Create a New Math Problem!* can be seen in Figure 4.

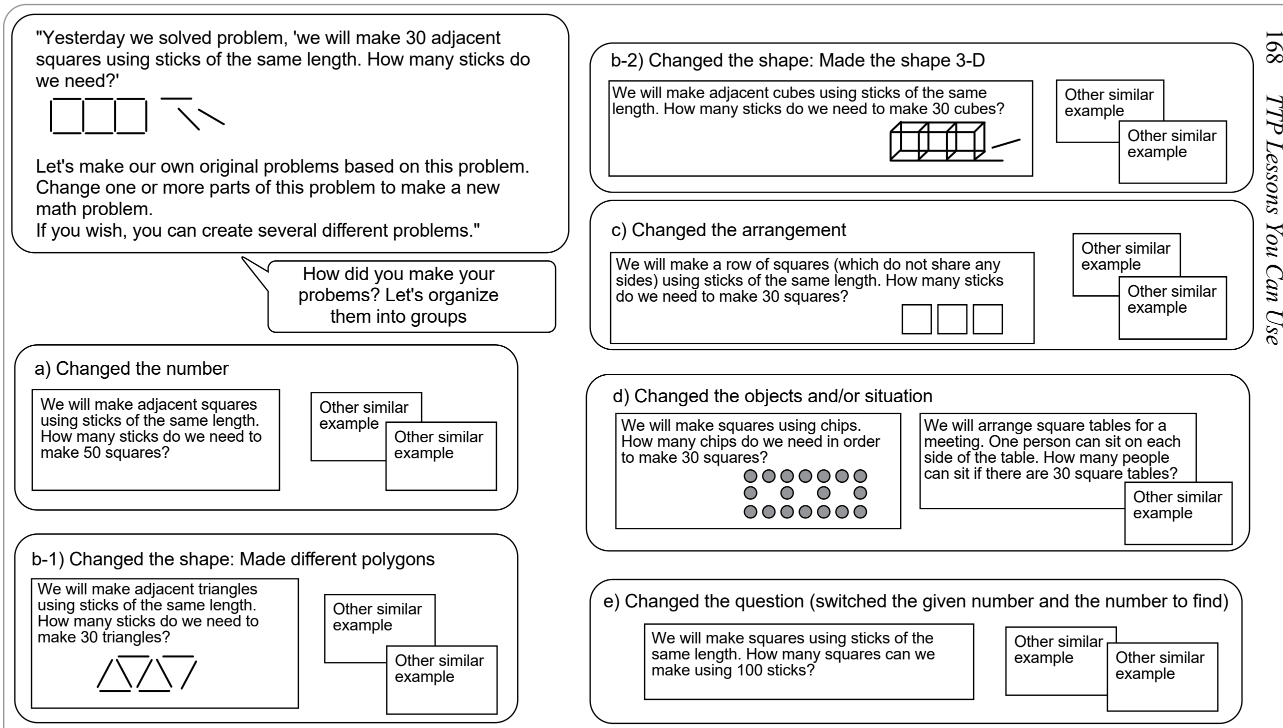


Figure 2.3.4.02 Neriage Map for "Let's create new math problems! A lesson from Mondai kara Mondai e [From Problem to Problem]"

Figure 4: *Neriage Map* of "*Let's Create a New Math Problem!* A lesson from Mondai kara Mondaie". This page is reproduced with permission from the publishers, who have licensed the use of a complete scanned page from the book.

In Chapter 4 of the book, the author connects research to practice by showcasing how CLR can transform school wide teaching practices. The process of CLR represented using Figure 5 exemplifies how research becomes practice. Teachers use instructional strategies such as anticipating student thinking, promoting student centred mathematical conversations, and designing cognitively demanding tasks not as abstract ideas but as tools for lesson design and reflection. CLR shifts professional development from isolated workshops to embedded, collaborative inquiry. Teachers learn from each other, from students, and from experts, creating a dynamic feedback loop that continuously improves teaching quality. Through CLR, schools can build a sustainable model where teachers are empowered as co-researchers, and student learning is at the centre of instructional design.

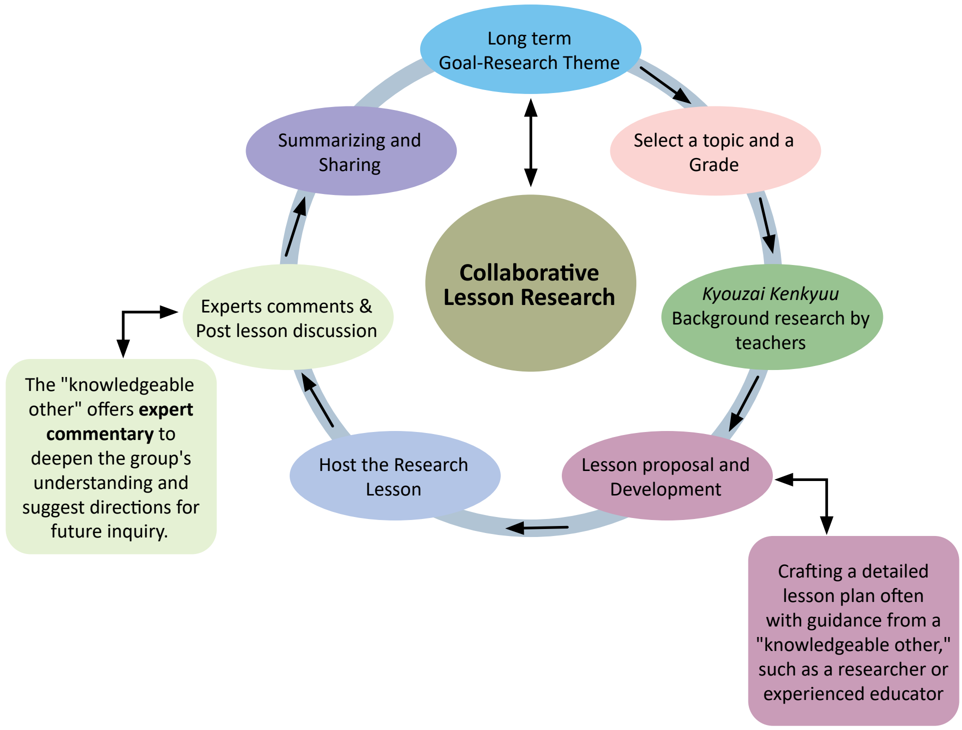


Figure 5: Steps in the CLR Cycle: from research theme to shared insights bridging classroom practice and professional inquiry.

Relevance to the Indian context and curriculum reforms

Teaching mathematics through problem-solving aligns closely with the principles of competency-based learning outlined in the National Curriculum Framework for School Education (NCF-SE) 2023. The book emphasizes student-led exploration, where learners engage with unfamiliar problems before receiving formal instruction. This fosters critical thinking, creativity, deep understanding and skills, which mirrors NCF-SE's focus on developing competencies rather than on rote memorization. Furthermore, the emphasis on reasoning, representation, and reflective discussion in the TTP framework supports NCF's vision of formative, feedback-driven assessment. Both ideologies advocate for empowering students to take ownership of their learning, promoting flexible pathways and holistic development. Together, they offer a powerful model for transforming mathematics education in Indian classrooms to be more engaging, equitable, and effective.

In the Indian context, the implementation of TTP faces several limitations, including large class sizes, limited opportunities for teacher collaboration, and exam-driven systems that restrict sustained professional dialogue. The implementation of CLR may also be challenging; however, the framework presents significant potential. If adapted thoughtfully, CLR can foster professional learning communities in which teachers progressively integrate problem-solving approaches into their classrooms while supporting one another.

Conclusion

This book brings together years of research, classroom experience, and thoughtful reflection. What makes the book truly special for me, is how it presents deep and meaningful ideas in a clear and practical way. It shows how problem-solving can help students think for themselves, work together, and understand math more deeply. Even though the themes are rich and complex, the author makes them easy to follow and apply. This book is a valuable guide for anyone looking to make math teaching more engaging, student-centred, and effective.

I also found the following resources useful for CLR and TTP, which interested readers may refer to:

1. The Lesson Study Group at Mills College https://bit.ly/4hlbRVP
2. The Lesson Study Group at Mills College and Teaching through problem solving https://bit.ly/47eF5Ru
3. Lesson Study Alliance. (2020). Lesson Study Resources. Retrieved from https://www.lsalliance.org/
4. Takahashi, A., & Yoshida, M. (2004). How Can We Start Lesson Study? Ideas for Establishing Lesson Study Communities. *Teaching Children* Mathematics, 10(9), 436-443.
5. Takahashi, A. (2008). Beyond Show and Tell: Neriage for Teaching Through Problem-solving - Ideas from Japanese Problem-solving Approaches for Teaching Mathematics. Paper presented at the 11th International Congress on Mathematics Education in Mexico.

Acknowledgement: The author gratefully acknowledges Routledge, Taylor & Francis Group for granting permission to reproduce a scanned page from the book in this article.

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Solution to Cross-Number Puzzle

on Page 34

	[1] 3	[2] 5	7			[3] 1	[4] 8	6	
[5] 4		1					2		[6] 8
[7] 1	2	4		[8] 5	6		[9] 4	3	0
9			[10] 3	2		[11] 4			2
		[12] 7				[13] 7	[14] 5		
		[15] 1	[16] 8				3		
[17] 9			4		[18] 3	4			[19] 8
[20] 5	0	[21] 2		[22] 6	1		[23] 3	0	4
0		4					2		0
	[24] 4	6	0			[25] 5	0	0	

Call for Articles!

At Right Angles is a quality resource dedicated to mathematics education in India's public education system. It is specifically designed for teachers and teacher educators at the foundational, preparatory, and middle school levels.

We invite articles from mathematics teachers, educators, practitioners, parents, and students. If you are looking for a platform to contribute articles that support and enhance the learning experience of mathematics particularly for students approximately in the age group 6-14 years , we welcome your submissions.

Suggested Topics and Themes

Submitted articles should focus on curricular content applicable to Classes 1-8 and could:

- Explain and illustrate themes and topics outlined in the National Curriculum Framework for School Education 2023 (NCF-SE 2023).
- Specifically address challenges discussed in the NCF-SE 2023.
- Be substantiated accounts of the history of mathematics or the history of mathematical thinking.
- Include innovative worksheets or methods to engage students in drill and practice.
- Describe real-life applications of mathematics relevant to the child's context.
- Describe interdisciplinary activities or projects.
- Review puzzles or games with a practical connection to the syllabus.
- Offer guidance on selecting relevant content, including online resources.
- Develop pedagogical strategies for foundational numeracy as well as computational thinking.
- Assist teachers in implementing differentiated teaching practices.
- Review of Teaching Learning Material (TLM) or describe how to use local context, and local TLM in the math class.
- Provide material to help students bridge gaps in conceptual understanding.
- Address issues in assessment.
- Suggest ways to identify and address misconceptions in mathematics learning.
- Offer a list of problems along with discussions on their solutions and problem-solving strategies that are not commonly found in textbooks.

In addition to full-length articles, we also welcome shorter pieces that can include a variety of engaging content. These could be reviews of books, mathematics software, or YouTube clips that explore mathematical themes. Other contributions can be 'proofs without words', mathematical paradoxes, 'false proofs', or creative expressions such as poetry, cartoons, or photographs with a mathematical theme. We also welcome anecdotes about a mathematician or interesting examples of 'maths in craft, movies, etc'.

Articles may be sent to
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2. **Catchy Title:** Title the article with an appropriate and catchy phrase that captures the spirit and substance of the article.
3. **Style:** Avoid a 'theorem-proof' format. Instead, integrate proofs into the article in an informal way.
4. **Balance:** Refrain from displaying long calculations. Strike a balance between providing too many details and making sudden jumps that depend on hidden calculations.
5. **Accessible language:** Avoid specialized jargon and notation that will be familiar only to specialists. If technical terms are needed, please define them.
6. **Use visuals:** Where possible, provide a diagram or a photograph that captures the essence of a mathematical idea. Never omit a diagram if it can help clarify a concept.
7. **Concise References:** Provide a compact list of references, with short recommendations.
8. **Exercises and Questions:** Make available a few exercises, and some questions to ponder either in the beginning or at the end of the article.
9. **Citation format:** Cite sources and references in their order of occurrence, at the end of the article. Avoid footnotes. If footnotes are needed, number and place them separately.
10. **Abbreviations and Acronyms:** Explain all abbreviations and acronyms the first time they occur in an article. Make a glossary of all such terms and place it at the end of the article.
11. **Labelling visual elements:** Label and number all diagrams, photos and figures included in the article. Attach them separately with the e-mail, with clear directions. (Please note: the minimum resolution for photos or scanned images should be 300 dpi).
12. **Precise references to visuals:** Refer to diagrams, photos, figures and tables by their numbers and avoid using references of these kinds: 'here', 'there', 'above', 'below', 'to the left', 'to the right'.
13. **Author Bio:** Include a high-resolution photograph (author photo) and a brief bio (not more than 50 words) that gives readers an idea of your experience and areas of expertise.
14. **British Spelling:** Adhere to British spellings – organise, not organize; colour not color, neighbour not neighbor, etc.
15. **Format for submission:** Submit articles in MS Word format or in LaTeX.

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At Right Angles

A RESOURCE FOR SCHOOL MATHEMATICS

Azim Premji University

ACTIVITIES THAT CELEBRATE MATHEMATICS

PADMAPRIYA SHIRALI

A publication of Azim Premji University

ACTIVITIES THAT CELEBRATE MATHEMATICS

In the Features section of this issue, we have looked at the Why and How of the celebration of a Mathematics Day. This pullout focuses on the **What:**

- What are the different strands which are featured in the stalls set up for the day?
- What are some tried and tested activities that work well for each strand?
- What are the materials needed?

Keywords: Mathematics day, exposition, content domains, visualisation, reasoning.

Here we give you the details arranged strand-wise.

Numbers

General advice: Having multiple sets of the material is useful at each of these stalls to accommodate more players at a time.

1. Prime Magic

Materials for each set: One grid card of (3×3) size with an extra column and row as shown in Figure 1. 1 set of number cards (1 to 9)

No. of players: 1

Task: Place the numbers 1, 2, 3,..., 9 on each square of the 3 by 3 grid so that each of the rows and columns adds up to a prime number.

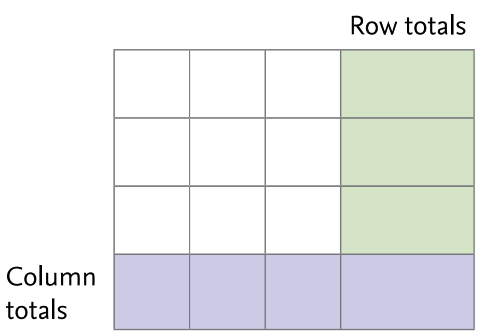

Figure 1

2. Sandwich Numbers

(Sourced from nrich.maths.org)

Materials: 2 sets of Number cards (1 to 7)

No. of players: 1

In this arrangement there is one number sandwiched between the two "1" cards, two numbers sandwiched between the two "2" cards, and three numbers sandwiched between the two "3" cards.

Make a complete sandwich with 1, 1, 2, 2, 3, 3, 4, 4.

Challenge: Make a sandwich using pairs of ones, twos, threes, fours, fives, sixes and sevens.

3. Number Tiklis!

Materials: Number pairs (chosen from numbers 1 to 9) written out on chits and three true statement cards that go with each pair. These can be pitched at varied levels, as shown in the example. (**Note:** The number chits are the tiklis (bindis). Self-adhesive post-it notes can be cut in interesting shapes to make these tiklis.)

No. of players: 2

Number pair example: 4 and 7.

Level 1 Statement card: The product of the numbers is 28.

Level 2 Statement card: When the numbers are arranged in fraction form, the fraction obtained is 12/21.

Level 3 Statement card: The sum of the squares of the numbers is 65.

The two players face each other. One number chit each is stuck on their forehead (by the student presenter at the stall) so that each player can see the number of the other person but does not know his/her own number.

The stall presenter selects any one of the statements (as per the mathematical knowledge of the players) and reads it out.

Each player has to work out what the number on their own tikli is, based on what is written on the other person's tikli and the statement that has been read out by the student presenter.

4. Get Close

Materials: 2 sets of Number cards (0 to 9), Six Instruction Cards with different target numbers

No. of players: 2

Operations allowed: Addition

Sample Instruction Card: 'Place the four digits to make 2 two-digit numbers, the sum of which is as close to 100 as possible.'

Note: As the phrase 'close to 100' may be interpreted as just the proximity to 100, numbers less than, equal to, or greater than 100 may be accepted. For example, 101 is a better answer than 96.

Both the players take turns to play the game. Let the first player pick 4 number cards without looking at the numbers on the card. The second player chooses an instruction card. The first player should try to come close to the instruction given on the card. The second player will see if he or she can come up with a better arrangement. (Zero cannot be a leading digit in forming two-digit numbers.)

Figure 2 shows the number cards picked and how they are to be placed.

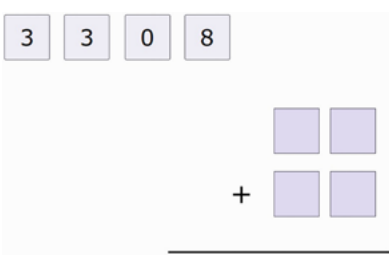

Figure 2

Higher level: The game can be made more challenging by having varied instruction cards and allowing for both addition and subtraction operations.

5. Squares all the Way: How long can you keep it going?

Materials: Paper, pencil

No. of players: No restriction

The presenter initially gives 4 numbers to be placed at the player's discretion at the four corners of a square. The player needs to write the difference between the larger and the smaller number of each pair that forms, at the mid-point of the corresponding side of the square. Join the midpoints to form a new square. Repeat the process till the difference becomes zero along all four sides.

The players can try with other numbers and see how long the process takes. They can predict choices which lead quickly to the answer.

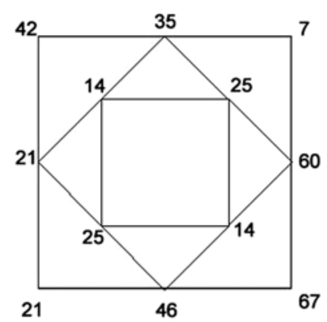

Figure 3

6. Challenge

Material: One flash card with the sequence 10, 15, 21, 4, 5

No. of players: No restriction

The presenter can pose the question 'What is special about this number sequence?'

10, 15, 21, 4, 5

If the participants are not able to spot any special feature, the presenter can explain.

'Each pair of adjacent numbers adds up to a square number.'

10 + 15 = 25 15 + 21 = 36 21 + 4 = 25 4 + 5 = 9

Task: The task is to now try to arrange the numbers 1 to 17 in a row in the same way, so that each adjacent pair adds up to a square number.

Mathematics through Visuals

1. Who's Who?

Groups of friends are represented by the graphs shown in Figures 4 and 5.

Each node represents a person. An edge (shown with a black line) joins two nodes (shown with yellow circles) if and only if those two people are friends.

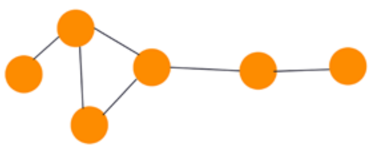

Figure 4

Can you work out who's who in Figure 4 using the clues below?

1. Anu has 3 friends: Bharath, Chandru, and Durga.
2. Bharath and Eela are both friends with Chandru.
3. Eela is Farha's only friend.

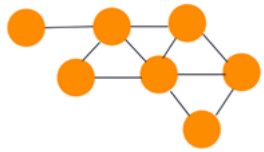

Figure 5

Figure 5 shows another graph depicting another group of friends.

Use the clues below to figure out who's who in Figure 5.

Bali and Clara are friends

Eesha and Clara are not friends

Bali is Fatima's only friend

Anu has more friends than anyone else

Dobe has three friends

Gopi and Dobe are not friends

Eesha has two friends.

2. Names Please!

Mintu, Bholu, Chotu, Gola, Ragi are here in Figure 6

Figure 6

Mintu and Ragi are smiling.

Ragi has big eyes.

Bholu and Mintu have big noses.

Chotu is sad.

3. Four Questions

Materials: Shape chart

No. of players: Either 2 or a group of 4 to 6

This activity is modelled on the same lines as the well-known game of 'Twenty Questions. '

In this game, Player 1 selects one card (Shape). The other players can ask questions to which the reply will be either 'Yes' or 'No'.

The players have to identify the shape in less than or equal to 4 questions.

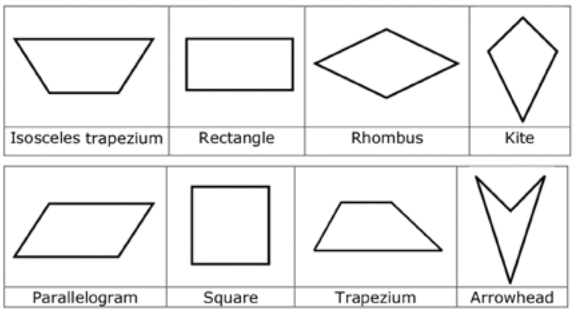

Figure 7

Shapes: Isosceles trapezium, Parallelogram, Rectangle, Square, Rhombus, Trapezium, Kite, Arrowhead.

Players need to figure out the questions that can help them to eliminate some of the choices.

Higher level: Can the players identify the shape after asking just three questions?

4. Planting Seeds!

Materials: 36 seeds and a (3 × 3) grid sheet with the middle square blanked out.

Plant the 36 seeds in the empty squares in the garden so that the top and bottom rows and left and right columns each add up to 18 seeds

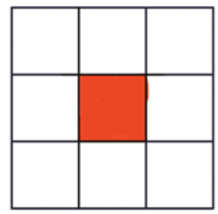

Figure 8

Plant 36 seeds in the empty squares in the garden so that the top and bottom rows and left and right columns each add up to 14 seeds.

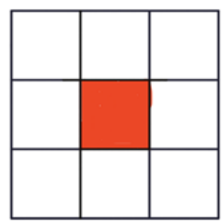

Figure 9

Measures and Estimation

1. Object Hunt

Materials: Balance, Rulers

How to Play

1. Challenge the participants to find objects that meet certain criteria. For example: An object that is approximately 15 cm long.
2. An object that weighs 50 g.
3. Check how close their estimate was with appropriate measuring instruments (A balance, a tape measure, etc.).

2. My Body!

Materials: Rulers and measuring tapes.

No. of players: 2

The presenter asks, 'Do you have any idea how long your head is?' (from the chin to the top of the head)

Let each player make a guess and then measure the length of each other's heads.

(**Note:** Data of this nature can be recorded and used later in a math class to get a sense of the variations of head length and to calculate the average.)

A study can be done to see if there is any difference between males and females. It is known that the length of a human head, from chin to the top of the head, is typically 8-9 inches (20-23 cm) for adults. Does the finding made by the students match this information?

3. Jugs!

There are many nice problems that involve unmarked jugs to measure out a required quantity.

Here is one such problem.

You have two jugs. One holds seven litres and the other holds five litres.

How can you measure out exactly 4 litres using these two jugs?

Figure 10

Geometry

1. How many Squares!

Materials: Picture card

No. of players: No restriction

How many squares are here?

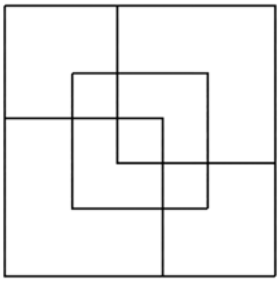

Figure 11

2. More Squares!

Materials: Tangram set

No. of players: 1

Can you make five differently sized squares by using all or some of the pieces of a tangram set?

(**Note:** Some solutions are given in Figure 27 at the end of the pullout.)

3. Sharing Land

Materials: Drawing of the shape on grid paper (1 drawing per person).

No. of players: No restriction

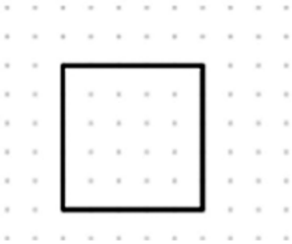

Figure 12

Figure 12 shows an old land sharing problem. Can you share this square piece of land into 5 equal parts in such a way that one person has no access to any of the four sides?

(**Note:** A solution is given in Figure 28 at the end of the pullout.)

4. Build

Materials: Models of 3D shapes, straws and connector set

No. of players: 2

3D shapes: It is only in the recent past that construction of 3D shapes has become part of the syllabus. Most adults would not have had a chance to build 3D shapes.

Display Models or pictures of the models. Provide straws and connectors and ask the participants to form regular 3D shapes such as a tetrahedron or an octahedron.

Games

1. Leapfrog

This is a well-known problem which works very well in any fun festival.

Materials: Cardboard strip with 5 circles (Figure 13a)

No. of players: 2 per strip

Figure 13a

The problem can start at a simple level with two brown frogs and two green frogs.

Setting: Series of 5 tiles with 2 green frogs on one end and 2 brown frogs on the other end with an empty circle in between. This is the start position. (Figure 13b)

Rules: A frog can slide from its position to the adjacent empty tile or jump over one frog to land in an empty tile. It can slide and jump both forwards and backwards. However, it cannot jump over more than one frog.

Figure 13b

Challenge: Can we swap the positions of the green and brown frogs? Did a frog have to move backward? What is the minimum number of moves needed to do so?

Extension: Try with an increased number of frogs (3 green and 3 brown). Can we swap the positions of the three green and three brown frogs? Did a frog have to move backward? What is the minimum number of moves needed to do so?

Figure 14a

Figure 14b

On a math festival day the aim might be to figure out if it can be done in the minimum number of moves. However, the problem can be explored further to notice a pattern in the sequence of moves. They could even attempt to explain the pattern and come up with a method for swapping frogs in the minimum number of moves.

On one occasion, when we presented this game using boards and counters, a group of senior students who were intrigued by the challenge, drew a figure on the floor outside the venue and used chappals and shoes to represent the two types of frogs! They ended up collecting a huge encouraging crowd around them as they tried to figure out the solution. Street Math!

2. Block!

(Sourced from nrich.maths.org)

Materials: Large figure card

No. of players: 2

This game is for two players. Each player needs two counters (or buttons, or stones) to put on the board below.

Place two stones at the top and two at the bottom as shown in Figure 15. (At the start of the next game the players should swap positions.)

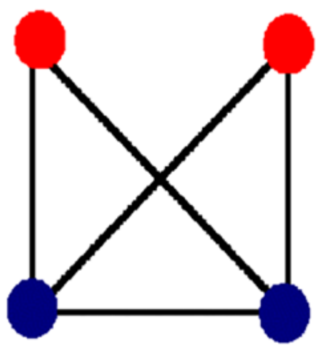

Figure 15

The players take turns at sliding one stone along a line to an empty spot. (So, the first move will always be to the middle.)

To win, you have to block the other player so he or she can't move.

In China this game is known as Pong hau k'i and in Korea it is called Ou-moul-ko-no.

3. Jump and Leave One

Materials: Numbered counters (1 to 10)

No. of players: 1

Arrange ten counters (1 to 10) in a triangle shape as shown in Figure 16. Remove coin numbered 9 to make a space. Now the remaining coins can jump over one another. Jump one coin over another coin into an empty space. You can now remove the coin you jumped over.

Challenge: Jump the coins one at a time so that there is only one coin left.

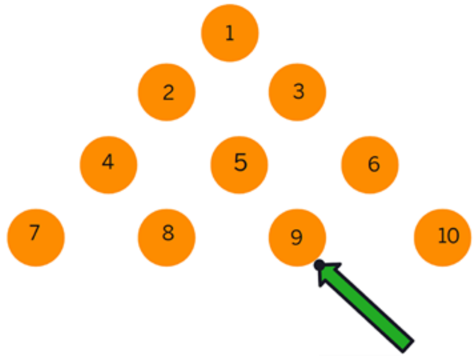

Figure 16

4. Stools Apart!

Materials: 8 stools to be arranged on the ground in the form given in Figure 17; big number cards.

This challenge is for a group of 8 students. Each student is given a number card (1 to 8) and they have to sit in such a way that no two consecutive numbers can be next to each other either horizontally, vertically or diagonally.

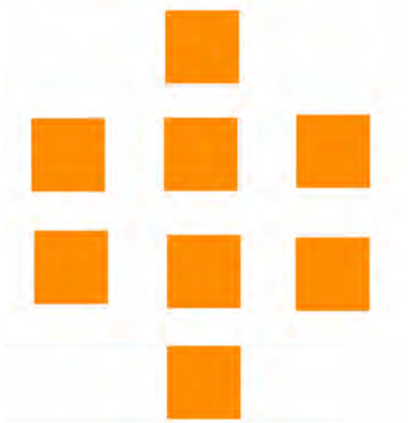

Figure 17

Puzzle space or workshop space

No Mathematics Day celebration can be complete without playing around with stimulating puzzles. One can use well known puzzles such as:

- Tangram
- Soma cube
- Brahma's tower of discs
- Pentominoes
- Matchstick puzzles

1. Dissection Puzzles

Shape dissections can be given with the actual outline of the assembled puzzle drawn on a paper.

But another task that is more challenging, is to give the pieces and specify the expected shape to be made. In this case, it is a T shape.

Figure 18

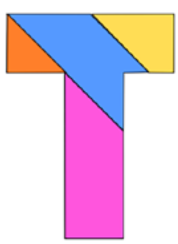

Figure 19

The pink, orange, blue and yellow shapes in Figure 19 have been fitted together to form a T. Can the same pieces be fitted together to form the shape shown in Figure 20?

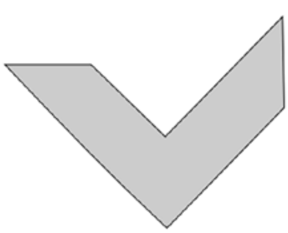

Figure 20

2. Tiling Challenges

Source: Polypad https://polypad.amplify.com/p

Materials: Tiling piece (at least 10 copies)

Provide several copies of an irregular tiling piece and ask the participants to tile a table with it.

Figure 21: The tile and the beginning of the tiling process.

3. River Puzzle and Bridge Puzzles

a) A man has a lion, a sheep and a basket of cabbages. He wants to take the animals and the cabbages to the other side of a river on a boat, but there is a problem. The man can carry with him only one of the three at a time on the boat.

- If he leaves the lion alone with the sheep at one side, the lion will eat the sheep.
- Similarly, the sheep will eat the cabbage if they are left alone.

How can he solve the problem?

b) There are 4 people (A, B, C, and D) who want to cross a bridge at night.

- A takes 1 minute to cross the bridge.
- B takes 2 minutes to cross the bridge.
- C takes 5 minutes to cross the bridge.
- D takes 8 minutes to cross the bridge.

There is only one torch with them and the bridge cannot be crossed without the torch. There cannot be **more than** two persons on the bridge at any time, and when two people cross the bridge together, they must move at the slower person's pace. Can they all cross the bridge in 15 minutes?

4. Mind Readers!

A mind reader stall holds great attraction for an audience. A mathemagician (dressed in an interesting attire) can perform a magic trick. The trick is based on a mathematical process that will produce a pre-determined result. The audience will be intrigued by the trick and may try to figure out how the trick works setting the audience on an unravelling mission.

Several puzzles of this type can be found easily. Here are some samples.

Mind-Reader Trick 1. Give the following instructions to the audience:

1. Pick any double-digit positive number.
2. Sum the digits.
3. Subtract the sum from your original number.
4. If the difference is a double-digit number, sum the digits again. Now, do you have a one-digit number?

Now pretend to think and then declare: 'Hmmmmm......, Let me see what your number is! It is 9!'

For a few two-digit numbers, Step 4 will be needed.

Can the student figure out why this trick works? It is simple algebra!

(**Note:** The solution is given at the end of the pullout.)

Mind Reader Trick 2: In this trick, the participant thinks of a number between 1 and 63. The student in the stall gives the participant the 6 cards shown in Figure 22, and the participant returns the card/s having her / his number to the student. The student figures out what the number is and reveals the number that was thought of.

1	3	5	7	9
11	13	15	17	19
21	23	25	27	29
31	33	35	37	39
41	43	45	47	49
51	53	55	57	59
61	63			

2	3	6	7	10
11	14	15	18	19
22	23	26	27	30
31	34	35	38	39
42	43	46	47	50
51	54	55	58	59
62	63			

4	5	6	7	12
13	14	15	20	21
22	23	28	29	30
31	36	37	38	39
44	45	46	47	52
53	54	55	60	61
62	63			

8	9	10	11	12
13	14	15	24	25
26	27	28	29	30
31	40	41	42	43
44	45	46	47	56
57	58	59	60	61
62	63			

16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	48	49	50	51
52	53	54	55	56
57	58	59	60	61
62	63			

32	33	34	35	36
37	38	39	40	41
42	43	44	45	46
47	48	49	50	51
52	53	54	55	56
57	58	59	60	61
62	63			

Figure 22

Why does it work?

(**Note:** The solution is given at the end of the pullout.)

5. Another enjoyable one from Jaadui Pitara, NCERT

The vanishing frog

Figure 23 (https://ncert.nic.in/dee/pdf/Jaadui_Pitara_User_Manual_English.pdf)

Workshops

Mathematics Days can also be used as a time for holding small workshops on art forms with mathematical connections. Some examples are Rangolis (Kolams), Islamic Art, Weaving and Knitting.

The school can invite a parent to hold a Rangoli making workshop.

Rangolis of varying complexity can be introduced.

Simple ones that can be made on a square dot grid using straight lines.

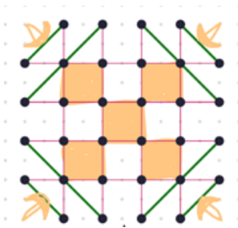

Figure 24

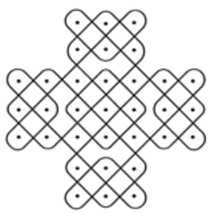

Figure 25

Slightly complex ones on dot paper (or on a black board), that are connected by a curved line that loops across dots in a continuous manner.

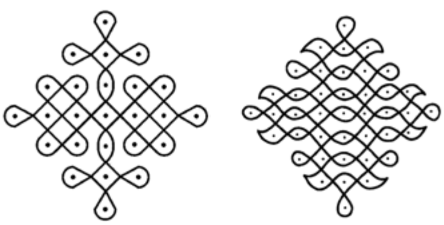

Figure 26

Finale: The day can end with a stimulating mathematics quiz or an engaging mathematical film. Some suggestions are given below:

1. Flatland https://share.google/images/qUevj2MkURQgE1GDv
2. Weird Numbers https://youtu.be/pSO66sL9SmY?feature=shared
3. Number Devil https://youtu.be/qJHc54IG5R8?feature=shared

Solutions to Selected Puzzles

More Squares: Problem 2 in the Geometry section

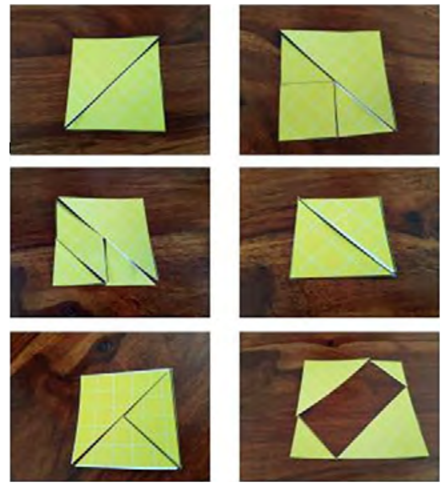

Figure 27

Land Sharing: Problem 3 in the Geometry section

Figure 28

Mind Reader Trick 1: The steps are as follows:

1. Let the chosen number be $10a + b$
2. Sum of the digits is $a + b$
3. $10a + b - (a + b)$ which will give $9a$
4. The digits of any multiple of 9 will always add up to 9
5. The answer will always be 9.

Mind Reader Trick 2: The numbers on the cards are written such that the student facilitator just needs to add the first numbers in the cards handed to him/her to guess the number thought of.

PADMAPRIYA SHIRALI

PADMAPRIYA SHIRALI is part of the Community Math Centre based in Sahyadri School (Pune) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. In the 1990s, she worked closely with the late Shri P K Srinivasan. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box.' She is currently part of the NCERT textbook development group. Padmapriya may be contacted at padmapriya.shirali@gmail.com